TOWARDS CALIBRATING THE COSMOLOGICAL COLLIDER

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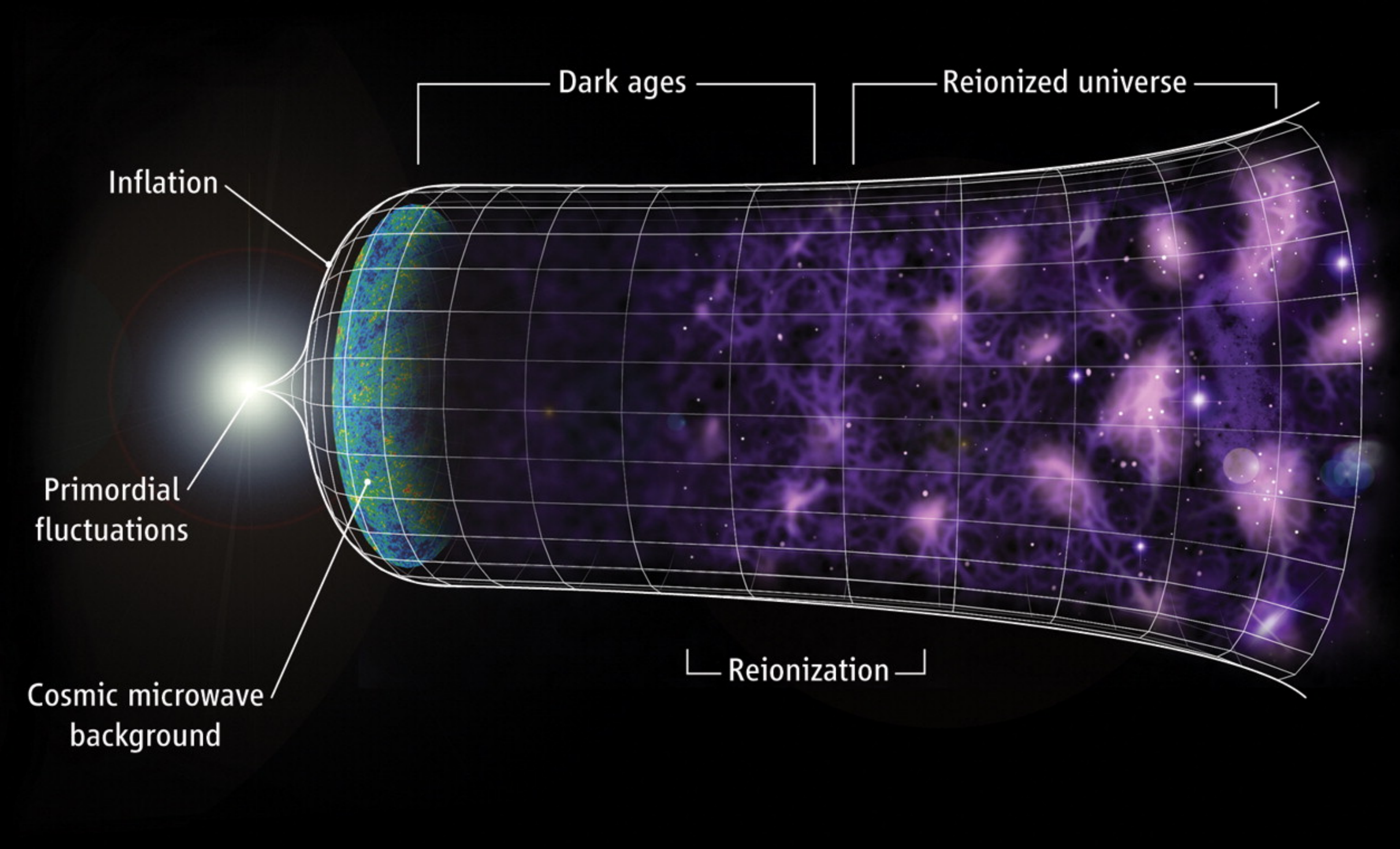
(CMSA Harvard)

Tsinghua | June 30, 2016

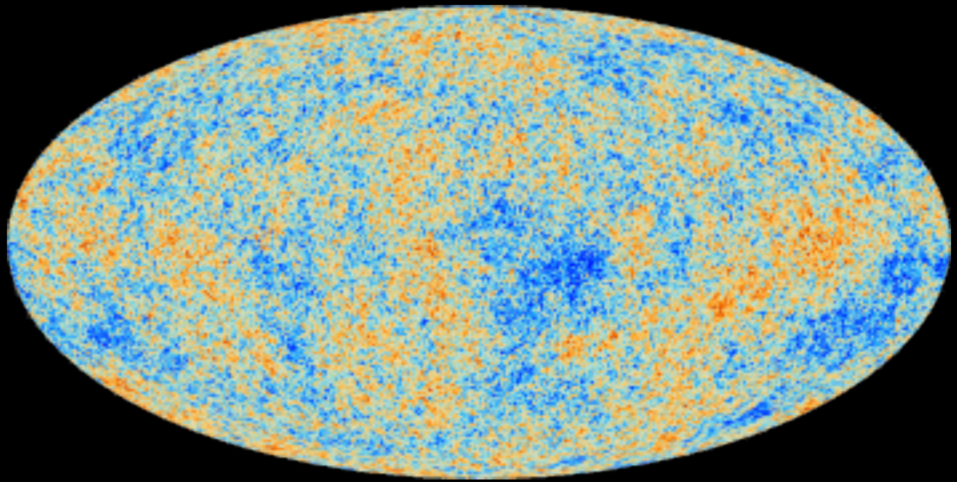
The Universe: A Historic Profile

- We are directly observing the history of the universe as we look deeply into the sky.



The Universe at Large Scales

- At ~10^4yrs the universe becomes so hot and dense that light cannot penetrate ⇒ CMB
- It is manifest from the observation of CMB that the universe is highly homogeneous and isotropic at large scales with ~10^{-5} fluctuations.
- But why?

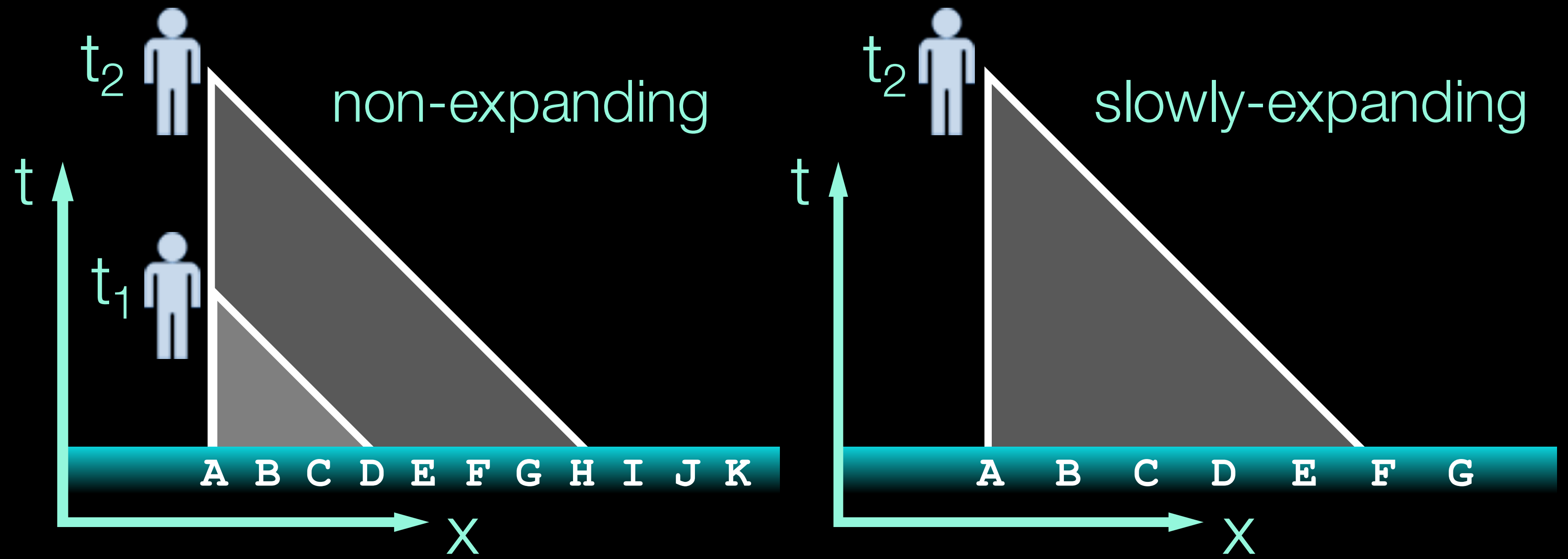


CMB sky (Planck 2015)

The Universe at Large Scales

- Big-bang cosmology:
  1) The universe has a starting point
  2) The universe expands "slower" than light cone

  1') Observable universe at a given time is finite
  2') Observable region is larger at late time



The Universe at Large Scales

- Big-bang cosmology:
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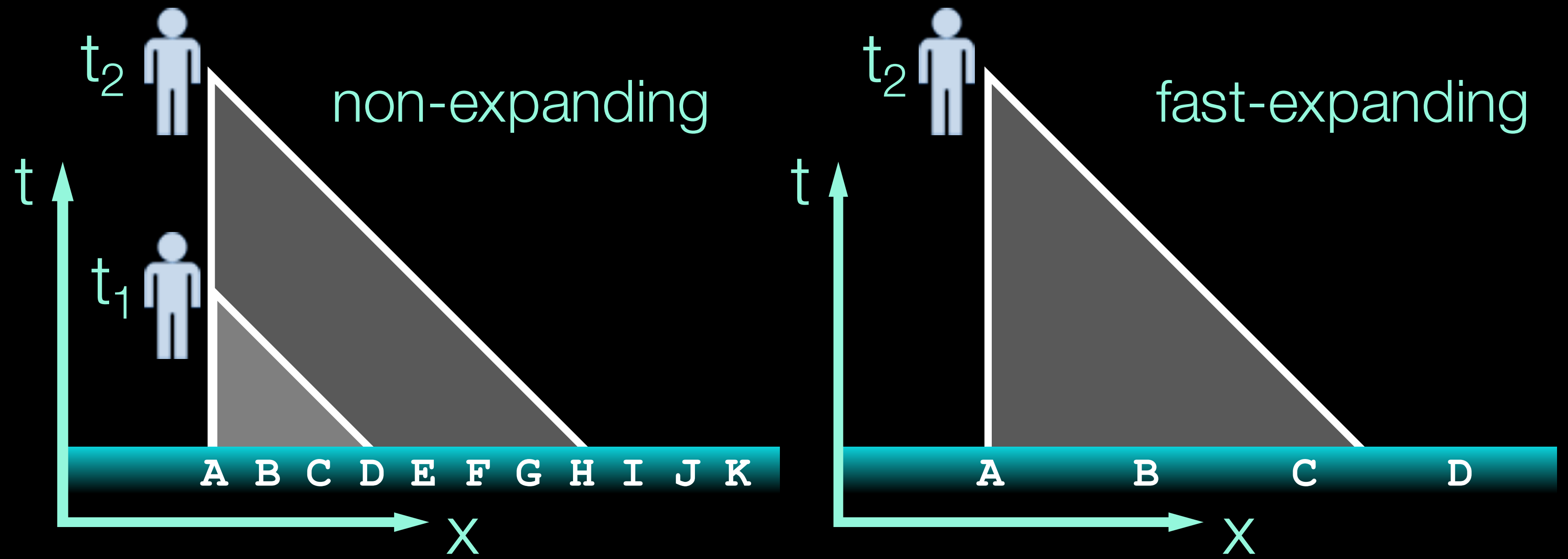
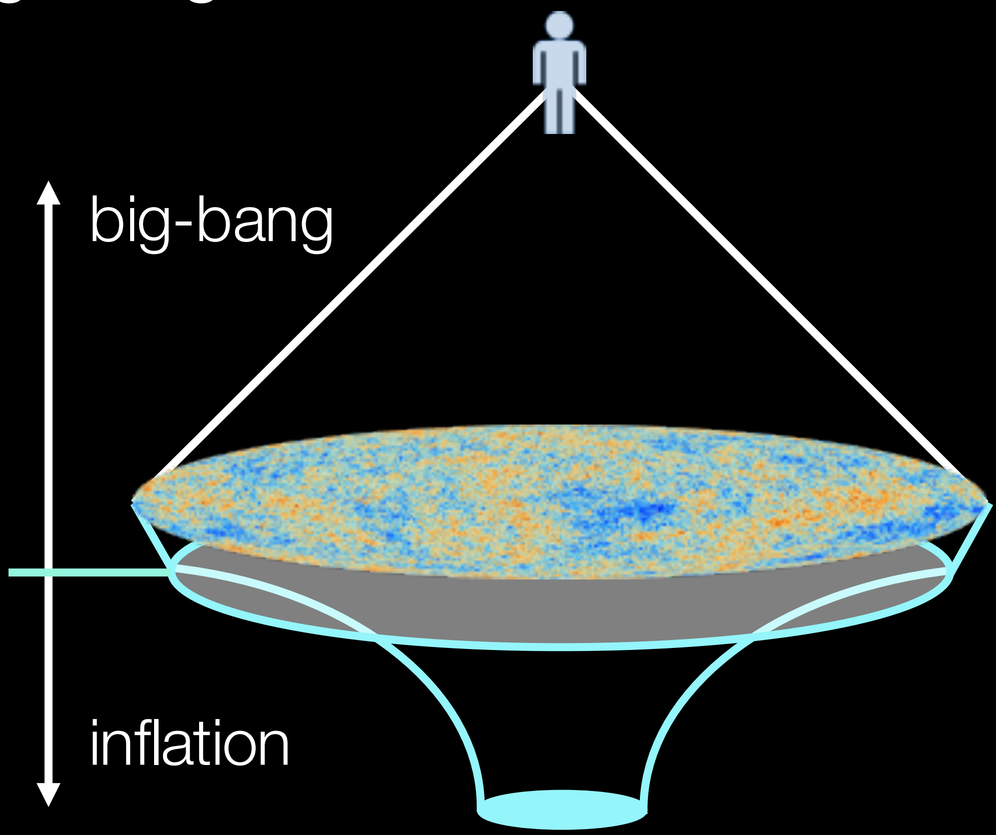


Cosmic Inflation

- Solution to horizon problem: A period of fast expansion before the thermal big-bang. $\Rightarrow$ Cosmic inflation.



- The expansion of universe can be parameterized by the scale factor $a(t)$. In simplest case $a(t) \sim e^{Ht}$.

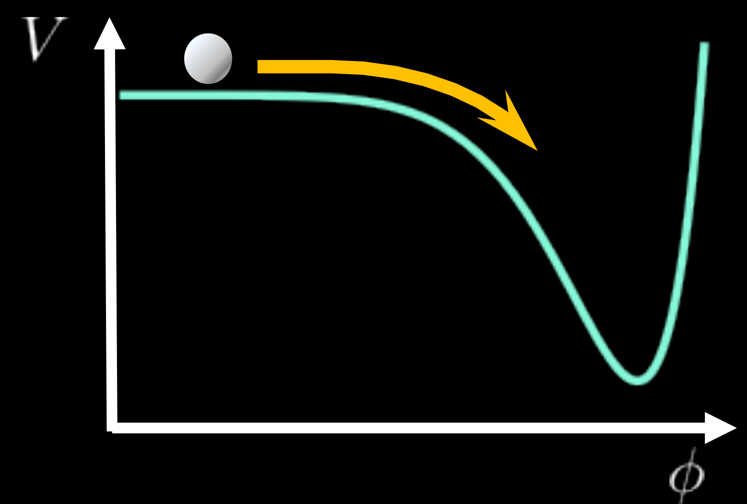
Cosmic Inflation

- A period of exponential expansion before the thermal big-bang era, driven by vacuum energy:

$$ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2$$

- Inflation ends with vacuum energy decaying to SM particles, thermalizing the universe. Reheating.

- Slow-roll paradigm:
  A scalar field – inflaton ϕ
  Very flat potential during inflation
  Fast decay during reheating



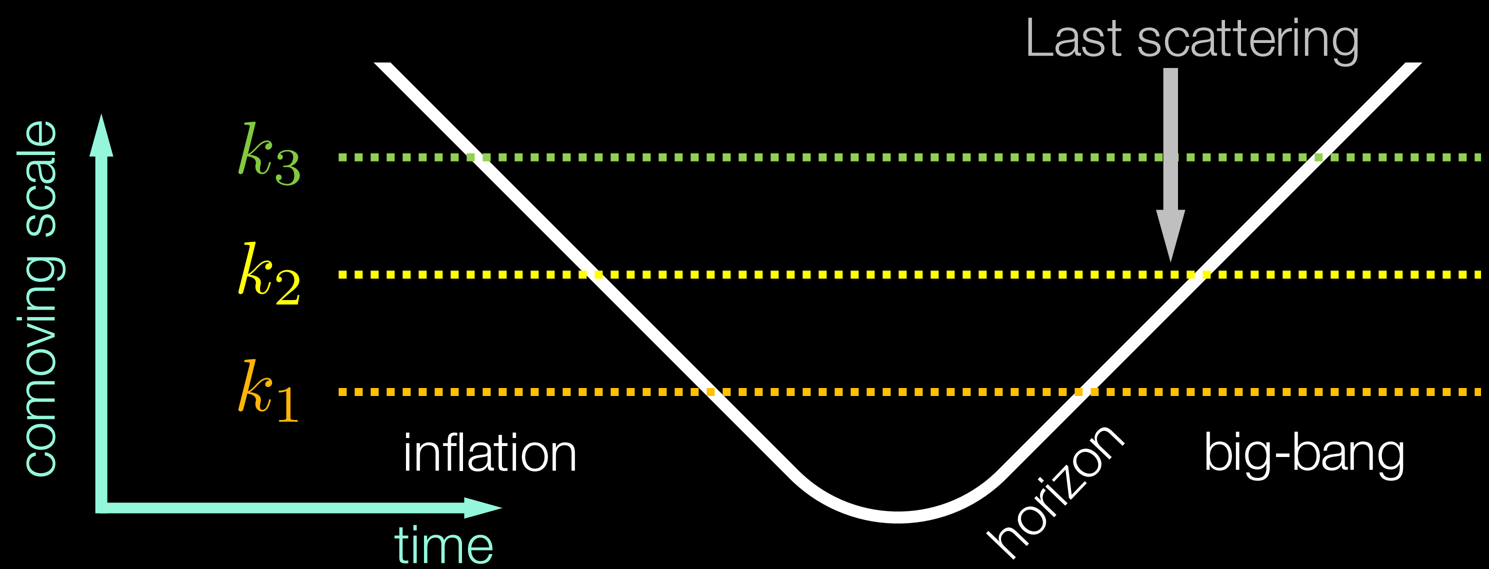
Cosmic Inflation

- Quantum fluctuations: the most fascinating aspect of cosmic inflation.

$$\mathrm{d}s^2 = N^2\mathrm{d}t^2 - \hat{g}_{ij}(N^i\mathrm{d}t + \mathrm{d}x^i)(N^j\mathrm{d}t + \mathrm{d}x^j)$$

$$\hat{g}_{ij} = e^{2(Ht+\zeta)}(\delta_{ij} + h_{ij})$$

scalar mode (ζ) tensor mode (h_{ij})



Cosmic Inflation

- Both scalar and tensor modes are frozen outside the horizon, and set the initial condition for the evolution of large scale structure as they reenter the horizon.

$$\zeta_k = -\frac{H}{\sqrt{4\epsilon k^3}}(1+\mathrm{i}k\tau)e^{-\mathrm{i}k\tau} \qquad \tau = -H^{-1}e^{-Ht}, \quad \tau \in (-\infty, 0)$$

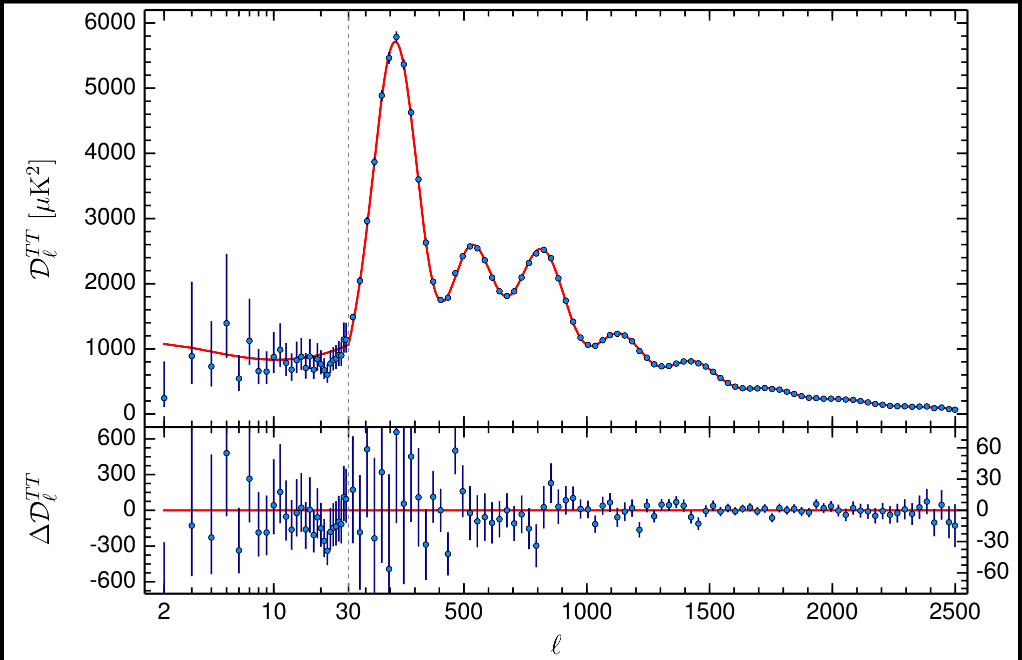
- Power spectrum:

Scalar mode:

$$\langle \zeta_{\vec{k}} \zeta_{-\vec{k}} \rangle$$

Tensor mode:

$$\langle h_{\vec{k}} h_{-\vec{k}} \rangle$$



(Planck 2015)

Cosmic Inflation

- Scalar power spectrum has been measured with impressive precision (almost but not exact scale invariant spectrum).

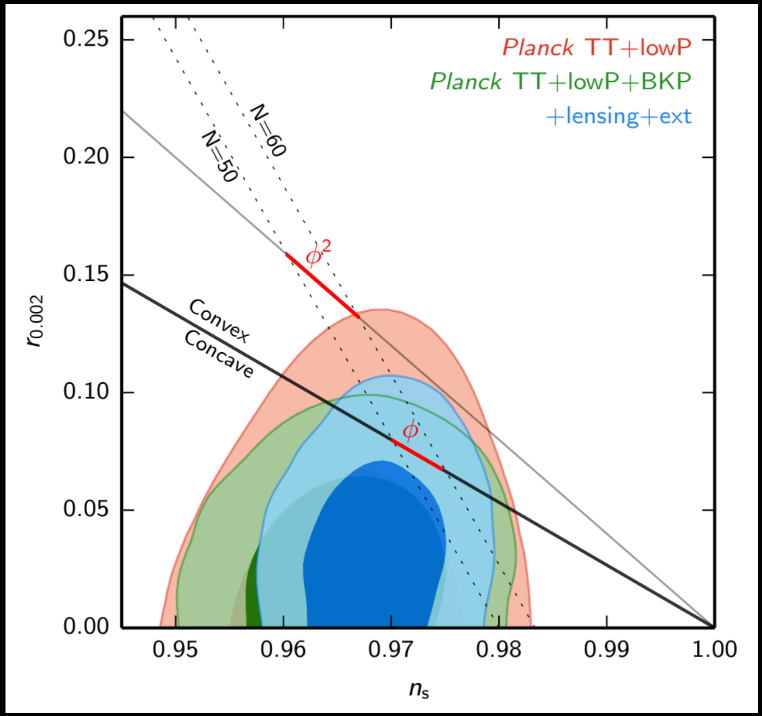
$$\zeta_k = -\frac{H}{\sqrt{4\epsilon k^3}}(1+\mathrm{i}k\tau)e^{-\mathrm{i}k\tau}$$

$$\langle \zeta_k^2 \rangle' = \frac{H^2}{4\epsilon k^3}$$

$$n_s \simeq 1 - 6\epsilon + 2\eta$$

$$r \simeq 16\epsilon$$

$$\epsilon = \frac{1}{2}(V'/V)^2, \qquad \eta = V''/V$$

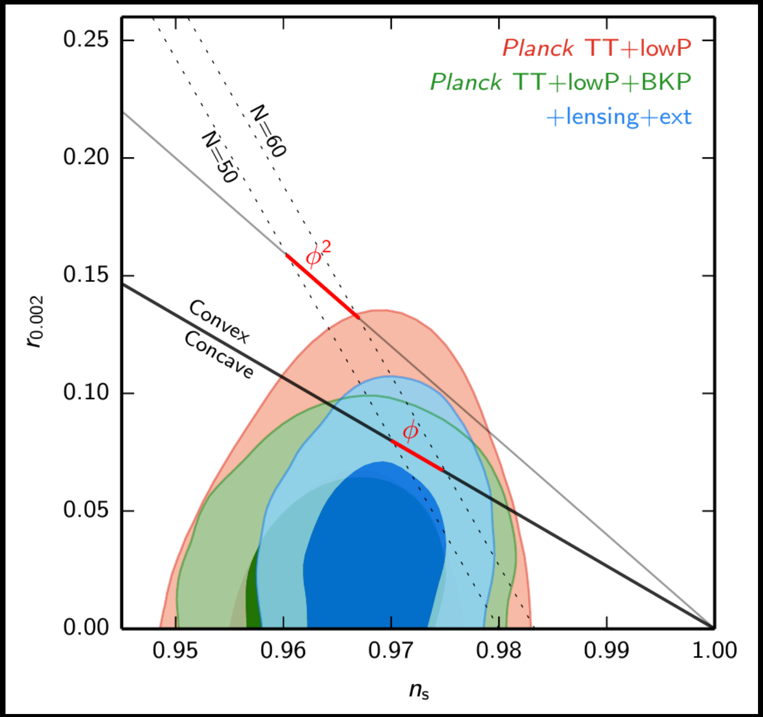


(Planck 2015)

Cosmic Inflation

- Scalar power spectrum has been measured with impressive precision (almost but not exact scale-invariant spectrum).

- Tensor mode (primordial gravitational wave) hasn't been observed yet. But the observation of tensor mode would provide very valuable information, e.g., quantum gravity, and the scale of inflation.



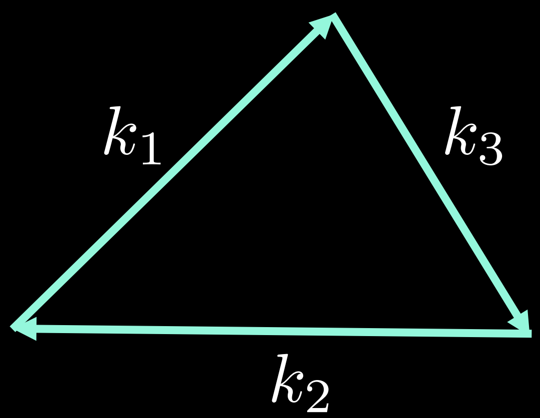
(Planck 2015)

Non-Gaussianity

- However valuable it may be, the information in power spectrum is limited. (~ scale invariance)
- A far more fascinating world beyond linear perturbation theory: Non-Gaussianity.
- Simplest possibility: 3-point function / bispectrum

$$\langle \zeta_1 \zeta_2 \zeta_3 \rangle, \quad \langle h_1 h_2 h_3 \rangle, \quad \cdots$$

- Scale invariant limit: power spectrum becomes a single number, while bispectrum is a function of the shape of triangles.



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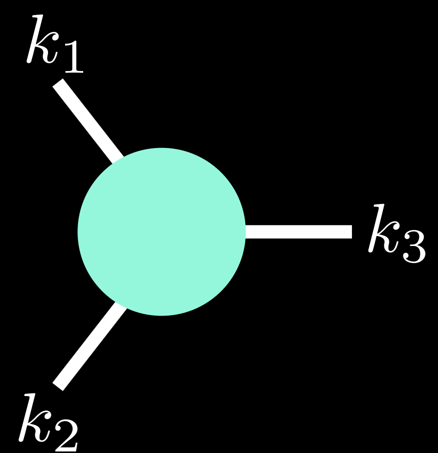
- Simplest possibility: 3-point function / bispectrum:

$$\langle\zeta_1\zeta_2\zeta_3\rangle, \quad \langle h_1 h_2 h_3\rangle, \quad \cdots$$

- Bispectrum probes interactions during inflation, and therefore possible new physics.
  $\Rightarrow$ Cosmological collider!

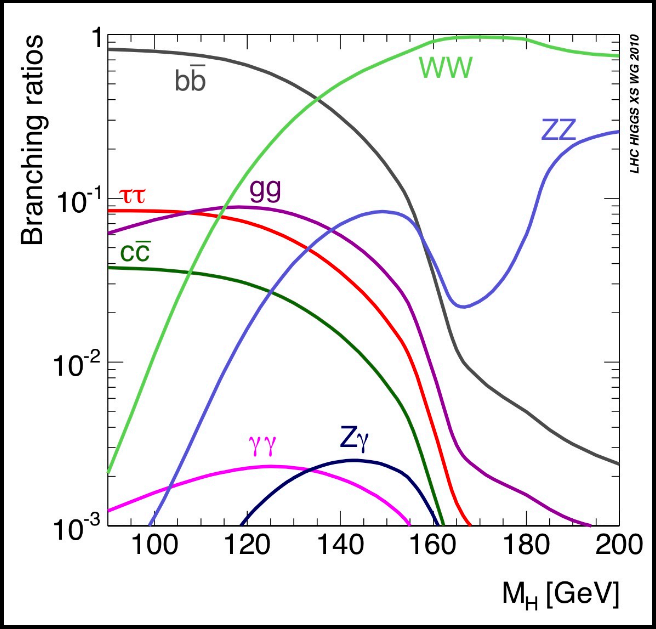


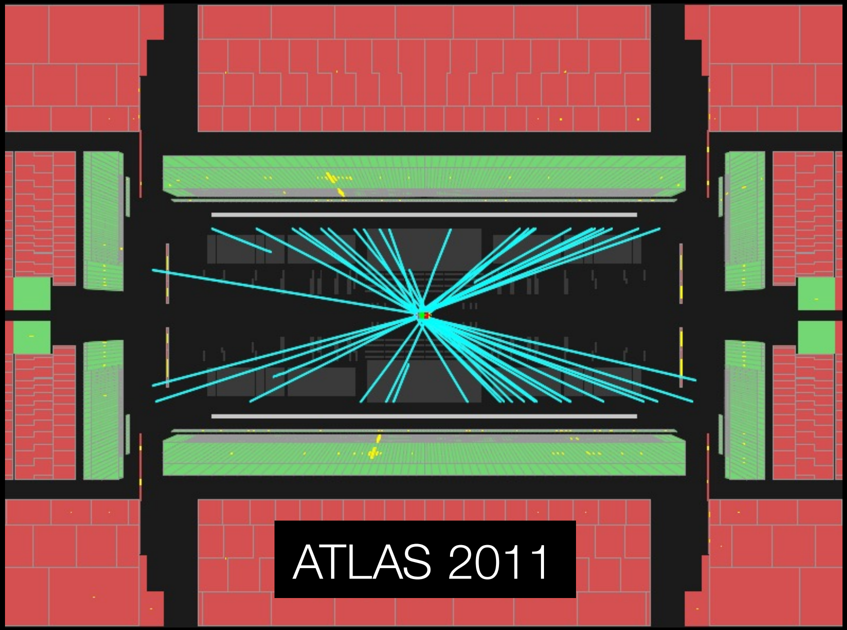
N. Arkani-Hamed, J. Maldacena, arXiv:1503.08043

Non-Gaussianity

- The most observable signals are usually not the most informative or distinguishable ones, and vice versa.

- Example: Decay of Higgs boson into two photons.





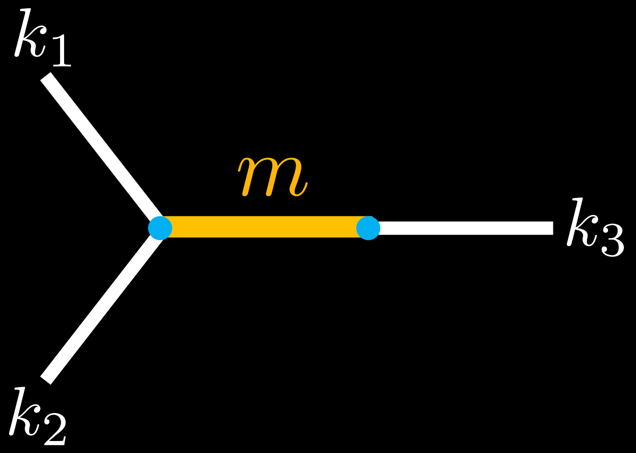
Non-Gaussianity

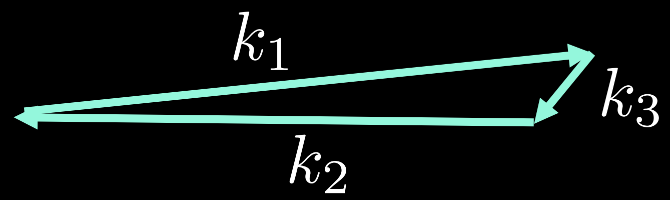
- Squeezed limit of bispectrum: $k_1 \simeq k_2 \gg k_3$

$$\frac{\langle\zeta_1\zeta_2\zeta_3\rangle}{\langle\zeta_1^2\rangle\langle\zeta_3^2\rangle} = \left[C(\mu)\left(\frac{k_3}{k_1}\right)^{\Delta_+} + C^*(\mu)\left(\frac{k_3}{k_1}\right)^{\Delta_-}\right]P_s(\hat{k}_1\cdot\hat{k}_3)$$

$$\mu = \sqrt{\frac{m^2}{H^2} - \frac{9}{4}} \qquad \Delta_\pm = \frac{3}{2} \pm \mathrm{i}\mu$$

- Quantum interference: massive particle & inflaton.





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$$\mu = \sqrt{\frac{m^2}{H^2} - \frac{9}{4}} \qquad \Delta_\pm = \frac{3}{2} \pm \mathrm{i}\mu$$

- $m > \frac{3}{2}H$: oscillations; Quantum Primordial Clock.

  X. Chen, M. H. Namjoo, Y. Wang, JCAP1602 (2016) 013

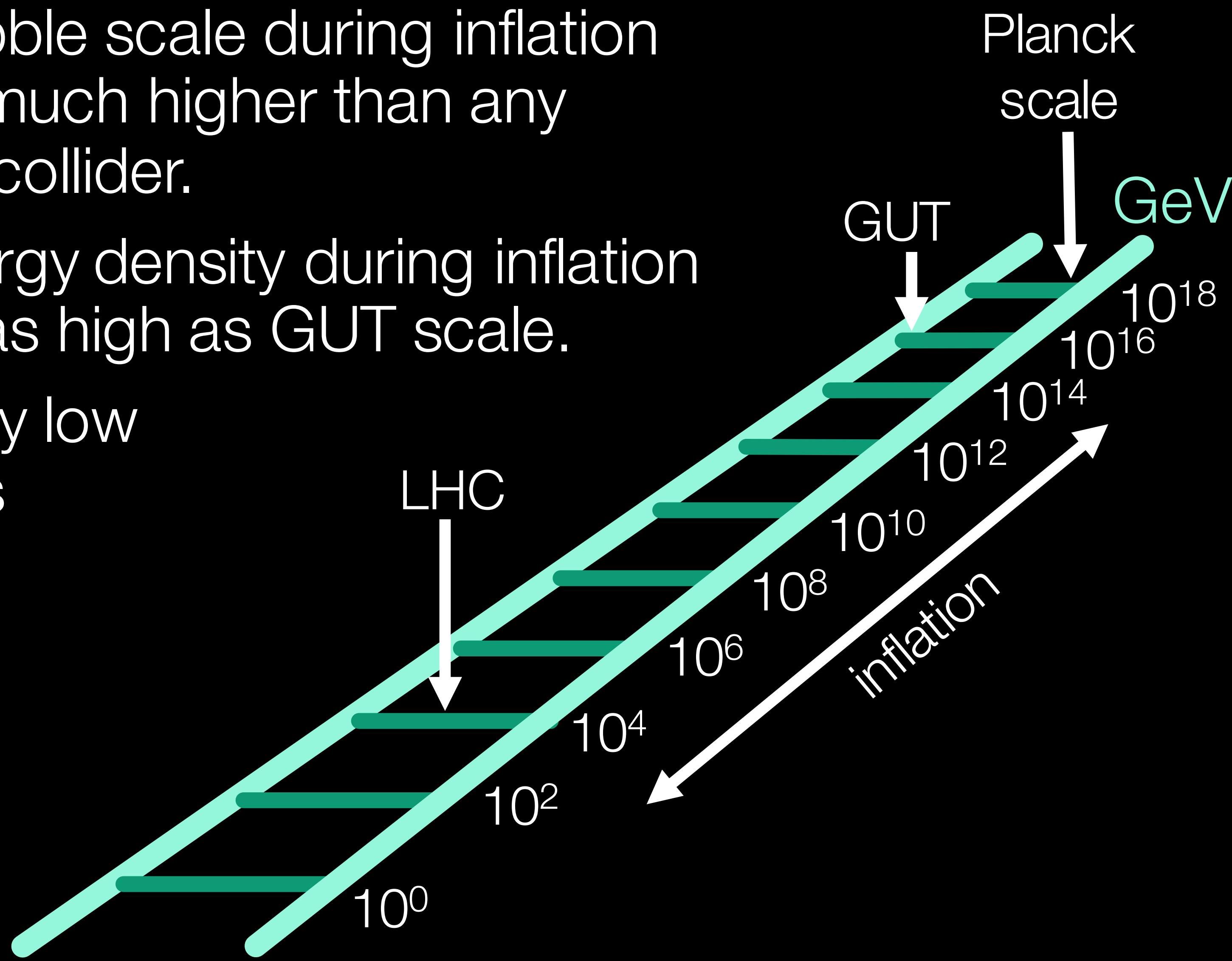
- $m < \frac{3}{2}H$: characteristic real power.

  X. Chen, Y. Wang, JCAP1004 (2010) 027

- Angular dependence: spin spectrum. (higher spins?)

Colliders: Artificial vs. Cosmological

- The Hubble scale during inflation can be much higher than any artificial collider.
- The energy density during inflation can be as high as GUT scale.
- Relatively low statistics



SM Fields in Inflation

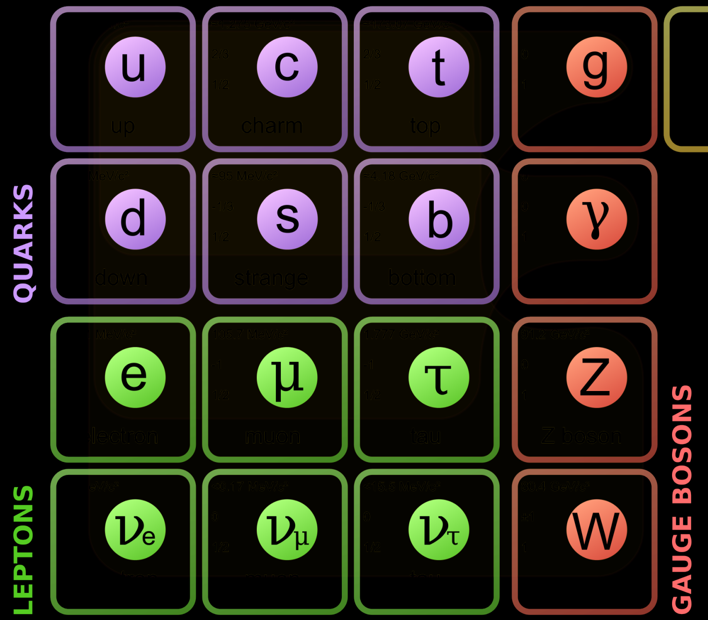
- Our knowledge of new physics is poor, but we do know Standard Model very well.
- It is important to calibrate the cosmological collider using knonw physics (SM) before exploring new physics.
- Inflaton must couple to SM fields to ensure efficient reheating.

X. Chen, Y. Wang, ZZX, arXiv:1604.07841

SM Fields in Inflation

- The mass spectrum of SM is determined by:
  1) Higgs couplings 2) Higgs VEV

$$M_X = g_X \langle \Phi \rangle$$



- When $\langle \Phi \rangle \neq 0$:
- Quarks, leptons, and W/Z are massive.
- Photon and gluon are massless.

SM Fields in Inflation

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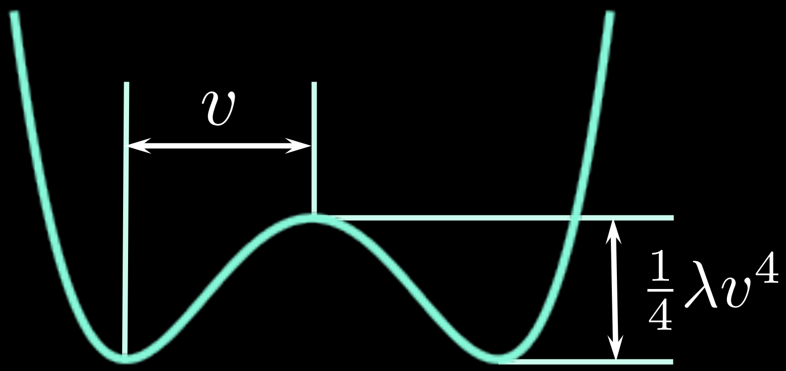
  1) Higgs couplings 2) Higgs VEV

$$M_X = g_X \langle \Phi \rangle$$

Higgs potential



$$v \simeq 246\text{GeV}, \quad \lambda \simeq 0.1$$

- Symmetry phase:

$$E \gg v \Rightarrow \langle \Phi \rangle = 0$$

- Broken phase:

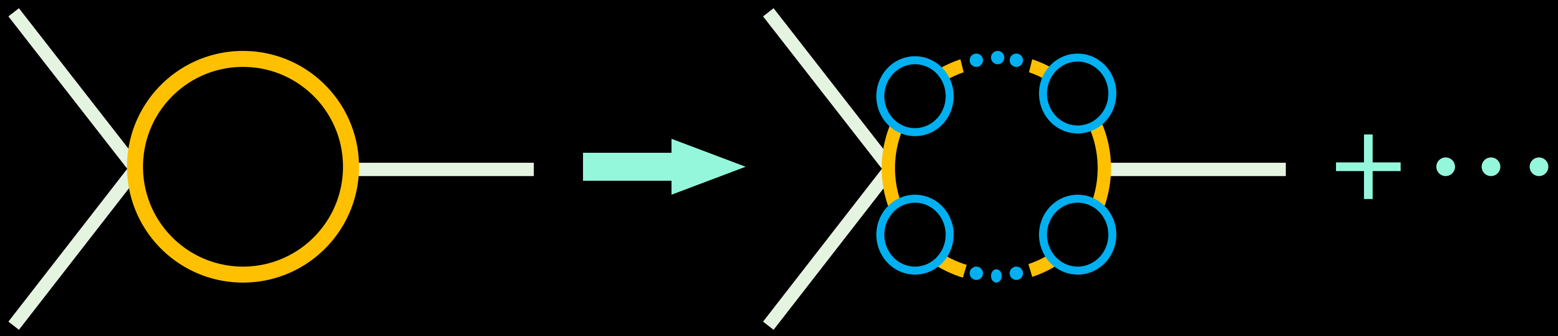
$$E \ll v \Rightarrow \langle \Phi \rangle = v$$

SM Fields in Inflation

- In inflation $E \sim H$, at least 3 possibilities:
- $H \ll v = \mathcal{O}(10^2 \mathrm{GeV})$: Usual electroweak broken phase. All fermions and W/Z massive.
- $H \gg v$: (Typical) Electroweak symmetry restored. All SM fields are effective massless at tree level.
- Higgs field itself is the inflaton (Higgs inflation): $\langle \Phi \rangle \sim \mathcal{O}(10^{19} \mathrm{GeV}) \gg H$, fermions and W/Z pick up huge and time-dependent mass.

SM Fields in Inflation

- Typical inflation: all SM fields are massless.
- No longer true when quantum corrections are included.



- For massless fields, higher order loops are as important as leading tree diagrams, because zero modes are strongly coupled in de Sitter space.

Loops in dS

- Analogy with thermal fields in flat spacetime.
- In massless $\lambda\phi^4$ theory with finite temperature T, the scalar field gets a thermal mass $m_{\rm th}^2 \sim \lambda T^2$
- The BD vacuum in de Sitter appears to be a thermal state for any time-like geodesic observer with temperature $T = H/2\pi$.
- The naïve guess for effective mass in de Sitter $m_{\rm eff}^2 \sim \lambda T^2 \sim \lambda H^2$ is incorrect.

Loops in dS

- This can already be understood using mean field theory, in which the effective mass is $m_{\text{eff}}^2 \sim \lambda\langle\phi^2\rangle$.
- For thermal $\lambda\phi^4$ in flat space, $\langle\phi^2\rangle \sim T^2$.
- In dS, two competing factors:
  - classical rolling-down along the potential: $\phi^2 \sim H/(\lambda t)$
  - quantum fluctuation: $\langle\phi^2\rangle \sim H^3 t$
- The equilibrium is reached at $t \sim (\sqrt{\lambda}H)^{-1}$

$$\Rightarrow \langle\phi^2\rangle \sim H^2/\sqrt{\lambda} \quad \Rightarrow \quad m_{\text{eff}}^2 \sim \sqrt{\lambda}H^2$$

in agreement with solving zero-mode partition function on Euclidean dS, S^4.

Loops in dS

- Photon mass?
- Thermal field theory in flat spacetime breaks Lorentz symmetry $\Rightarrow$ "Electric mass" is allowed for photon.
- The gauge invariant mass corresponds to the nontrivial Wilson loop winding around compact imaginary time direction.
- BD vacuum is dS invariant. The Euclidean version doesn't allow nontrivial Wilson loop
  $\Rightarrow$ No gauge-invariant photon mass?

Loops in dS

- Subtleties of Wick rotation.
- No IR divergence or late-time divergence can occur in Euclidean dS (compact and bounded by Hubble radius).
- For studies of inflation, it is conceptually safer to work with real-time Schwinger-Keldysh formalism without Wick rotation.
- SK formalism: calculating in-in correlators using Feynman diagrams:

$$\langle \text{in}|\mathcal{O}\mathcal{O}|\text{in}\rangle = \sum_{\text{out}} \langle \text{in}|\mathcal{O}|\text{out}\rangle\langle \text{out}|\mathcal{O}|\text{in}\rangle$$

Higgs again!

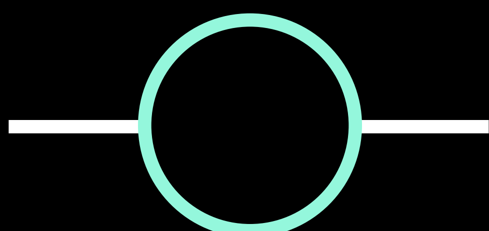
- The late-time divergences all come from Higgs loops.
- Intuitively, a massless scalar field is frozen outside horizon and leads to late-time divergence. Massless fermion and vector boson dies away
- The action for massless fermion and vector boson is (classically) Weyl invariant.
- Power counting of scale factor: “safe interaction” and “dangerous interaction” à la Weinberg.

Late-Time Divergence

- Two different types of loops: similar in UV but qualitatively different in IR / late-time.

$$\sim \int \mathrm{d}^4x\, G(x,x)$$

- The loop integral itself is time independent by de Sitter symmetry; mass insertion.



$$\sim \int \mathrm{d}^4x\mathrm{d}^4y\, G^2(x,y)$$

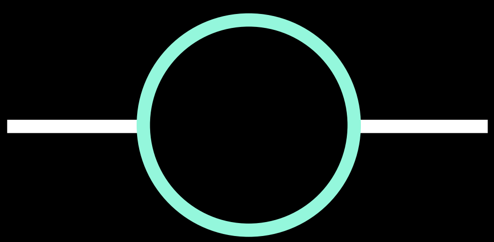
- The leading order late-time divergence is from the region where all momenta go soft.

Late-Time Divergence

- Two different types of loops: similar in UV but qualitatively different in IR / late-time.

$$\sim \int \mathrm{d}^4x\, G(x,x)$$

- $\propto \ln(-k\tau)$ when $\tau \to 0$.



$$\sim \int \mathrm{d}^4x \mathrm{d}^4y\, G^2(x,y)$$

- $\propto \ln^3(-k\tau)$ when $\tau \to 0$.

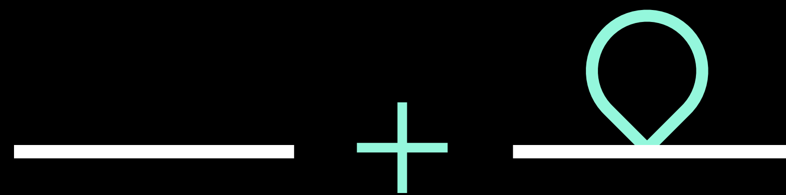
$$\tau = -H^{-1}e^{-Ht}$$

$$\tau \in (-\infty, 0)$$

Late-Time Divergence

- One loop results and Resummation: $\lambda\phi^4$ example.

$$-\mathrm{i}(1+\delta_{\text{1-loop}})G_S = \frac{H^2}{2k^3}\left[1+\frac{\lambda H^2}{2(2\pi)^2 m^2}\log(-k\tau)\right]$$



- Resummation by Dynamical Renormalization Group

$$G_S\Big|_r = \frac{H^2}{2k^3}(-k\tau)^{\lambda H^2/2(2\pi m)^2}$$

Resummed Propagators

- Resummed scalar propagator:

$$G_S\Big|_r = \frac{H^2}{2k^3}(-k\tau)^{\lambda H^2/2(2\pi m)^2}$$

- Compare it with massive propagator:

$$G_S(m) \propto (-\tau)^{2m^2/3H^2}$$

- Effective mass: $m^2 = \dfrac{\sqrt{3\lambda}H^2}{4\pi}$

- Partial resum: order 1 uncertainty; exact in large N limit; canceled out in the ratios of spectrum.

Resummed Propagators

- Fermions and vector bosons.
- Fermion's propagator receives late-time divergence from Yukawa interaction $y\sqrt{-g}\phi\bar{\psi}\psi$:

$$-\mathrm{i}G_F = \frac{H^3\tau^3}{2k}\left[k\gamma^0 - \exp\left(\frac{y^2}{(2\pi)^2}\log^3(-k\tau)\right)\mathbf{k}\cdot\boldsymbol{\gamma}\right]$$

- Charged scalar corrects "photon" propagator with both types of diagrams:

$$-\,\mathrm{i}(G_V)_{00} = \frac{1}{2k}(-k\tau)^{6e^2H^2/(2\pi m)^2}\eta_{00}$$

$$-\,\mathrm{i}(G_V)_{ij} = \frac{1}{2k}\exp\left[\frac{e^2}{24(2\pi)^2}\log^3(-k\tau)\right]\eta_{ij}$$

Resummed Propagators

- Photon:

$$-\mathrm{i}(G_V)_{00} = \frac{1}{2k}(-k\tau)^{6e^2H^2/(2\pi m)^2}\eta_{00}$$

$$-\mathrm{i}(G_V)_{ij} = \frac{1}{2k}\exp\left[\frac{e^2}{24(2\pi)^2}\log^3(-k\tau)\right]\eta_{ij}$$

- Compare it with massive photon:

$$G_V(M) \propto (-\tau)^{2M^2/H^2}$$

- Photon gets a nonzero “electric mass”:

$$M^2 = \sqrt{3}e^2H^2/(\pi\sqrt{\lambda})$$

- Only true when $e^2/\sqrt{\lambda} \ll 1$.

Resummed Propagators

Self-consistency check:

- finiteness of resummed propagator
  $\Rightarrow$ the coefficient of leading late-time divergence should be positive.
- Quantum corrections don't generate oscillatory behavior (clock signal), at least in perturbation theory.
- Mass relation of general spin s>0,

$$\mu = \sqrt{\frac{m^2}{H^2} - \left(s - \frac{1}{2}\right)^2}$$

$\Rightarrow$ scalars and photons can receive nonzero mass from loops while spin-1/2 fermion cannot.

SM Spectrum during inflation

- Typical Inflation: characteristic power dependence on momentum ratio in squeezed limit of bispectrum, from Higgs boson and massive gauge bosons.

- Very low scale inflation: Standard clock signals with full SM spectrum in the broken phase.

- Higgs inflation: Clock signals from relatively light fermion loops, with time-dependent mass.

$\Rightarrow$ A possible new way of model discrimination.

X. Chen, Y. Wang, ZZX, in preparation

Summary: Take-Home Messages

1. Inflation: the large scale structure of our universe has a quantum origin!

2. The primordial non-Gaussianity can be utilized as a super particle collider: Cosmological Collider

3. SM has nontrivial dynamics during inflation and can be used to calibrate the cosmological collider.

THANK YOU