Cosmic Inflation & Particle Physics

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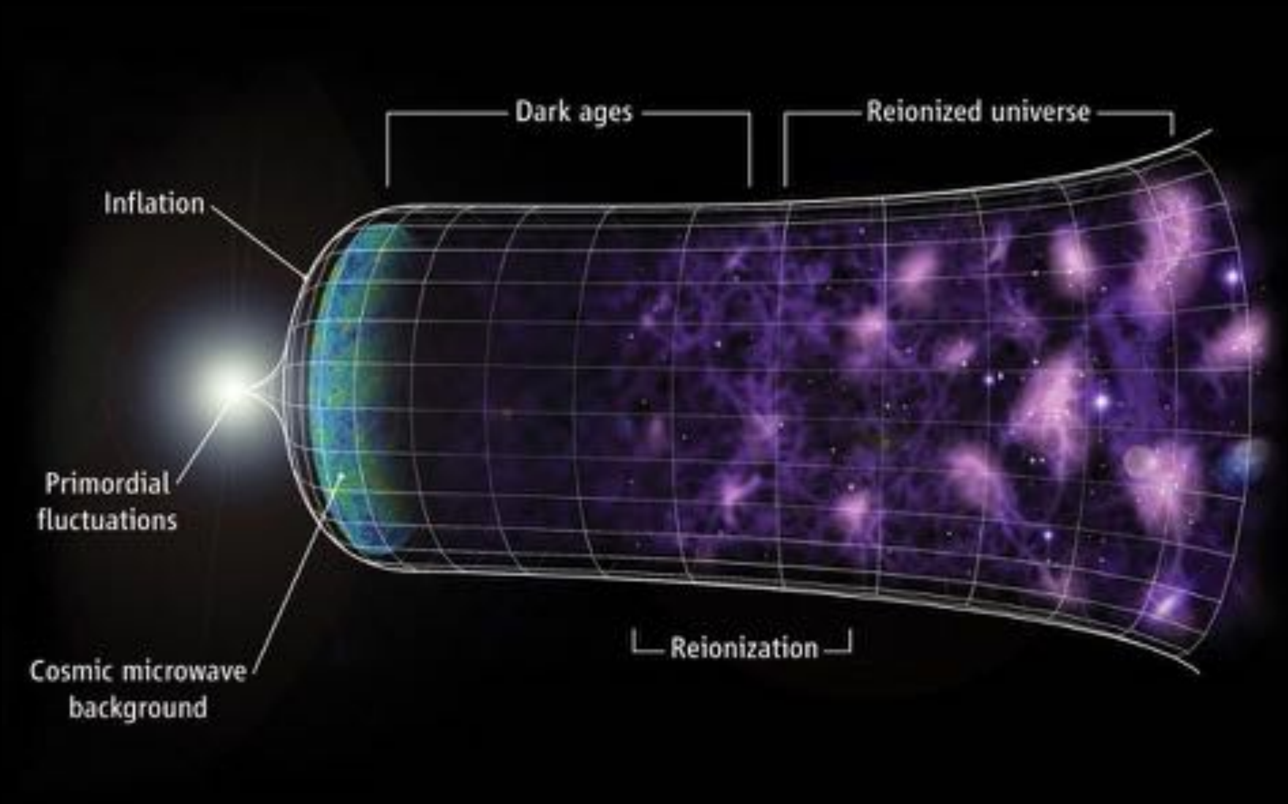
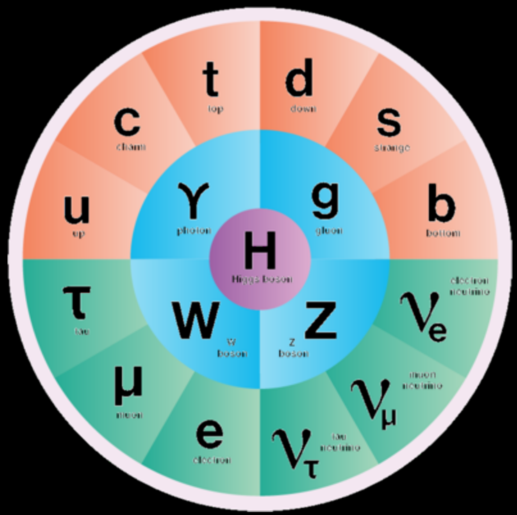
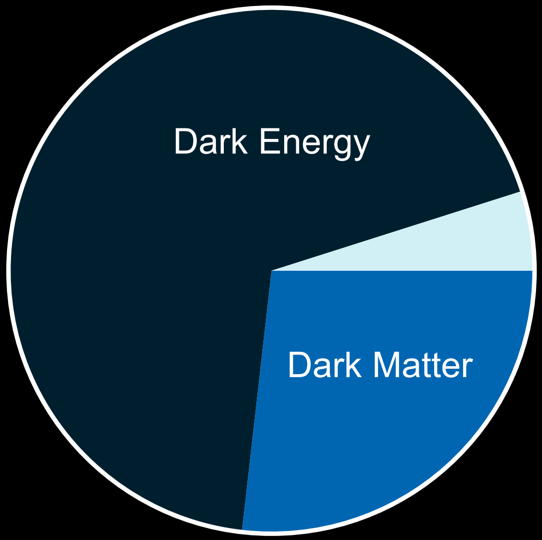
Our universe: an historical view



Our universe: an reductionistic view



Prof. Zhu's lecture



Prof. An's lecture

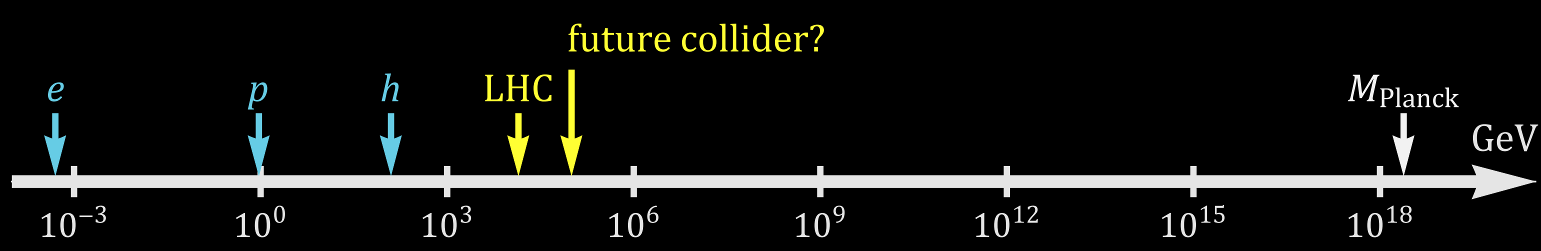
Our universe: an "emergent" view

Natural units

$$c = \hbar = k = 1$$

Only one unit is needed. For HEPer, it is GeV

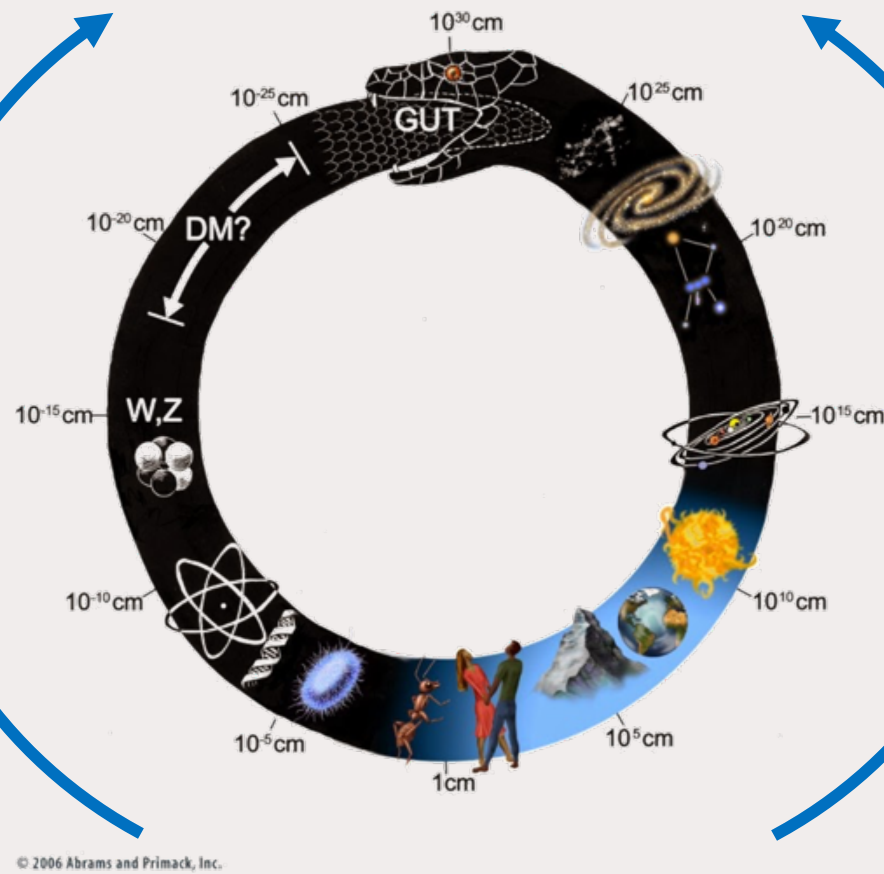
1GeV ~ Proton's mass   1g ~ 10^{23} GeV



A cosmic Ouroboros

Prof. Chen's lecture

Microscopic
Quantum Physics
Particle Physics

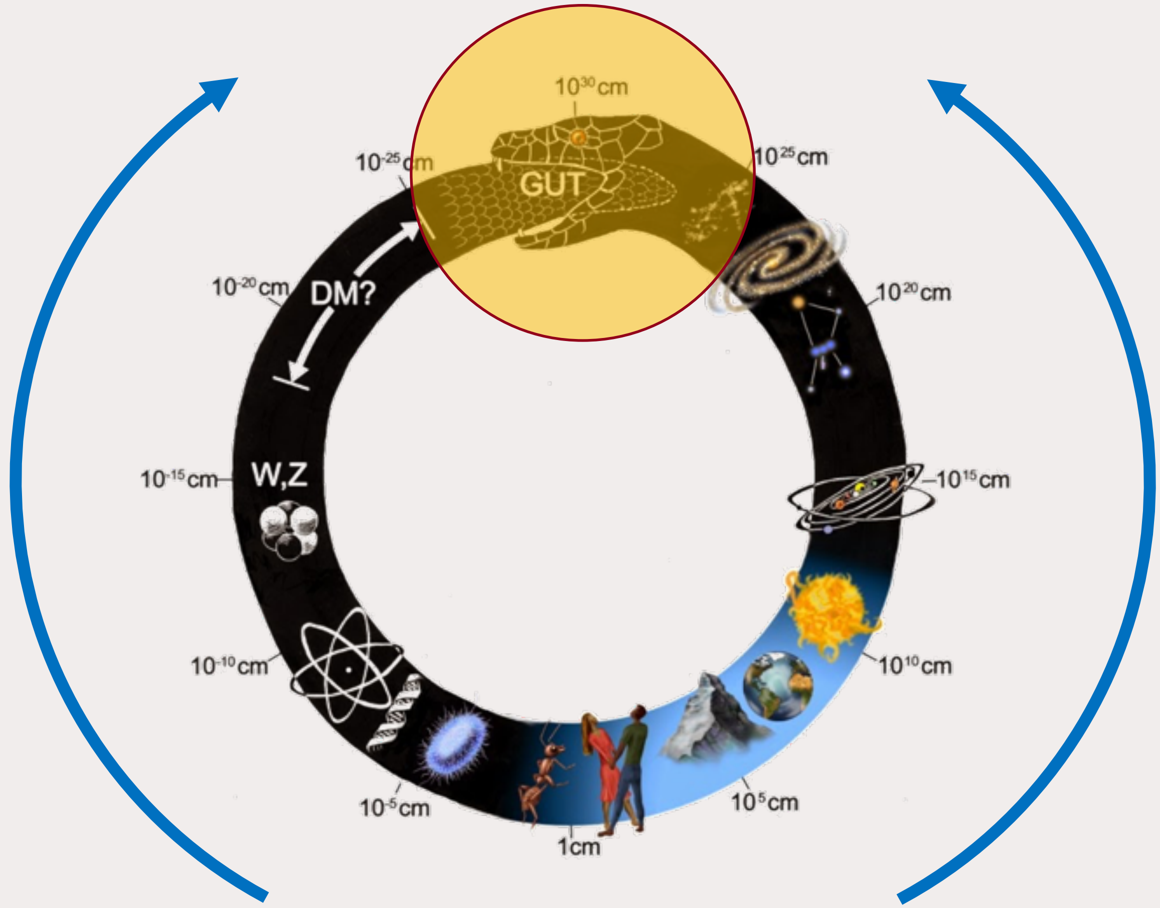


Macroscopic
General Relativity
Cosmology

A cosmic Ouroboros

INFLATION

Microscopic
Quantum Physics
Particle Physics



Macroscopic
General Relativity
Cosmology

Outline

1. Why we need inflation
2. Inflation dynamics and models
3. Cosmological collider physics

Useful refs

Cosmology in general

Dodelson: *Modern Cosmology*

Inflation

Baumann & McAllister: *Inflation and String Theory* [1404.2601]

Baumann's two TASI lectures: arXiv: 0907.5424; 1807.03098

Non-Gaussianity

Chen: 1002.1416

Wang: 1303.1523

Cosmological collider physics

Arkani-Hamed & Maldacena: 1503.08043

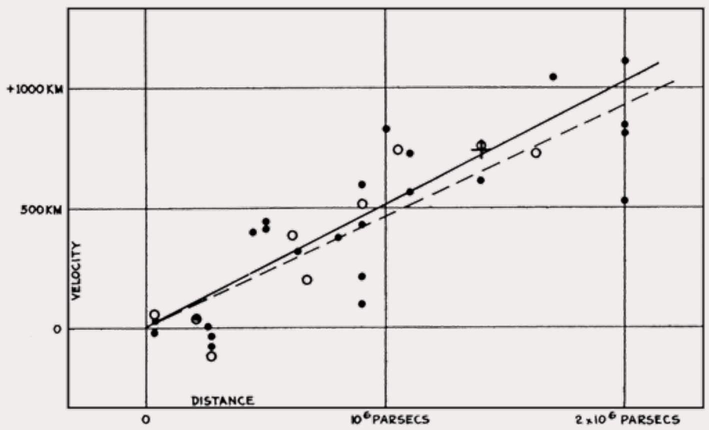
Chen, Wang, Xianyu: 1703.10166

Wang, Xianyu: 1910.12876

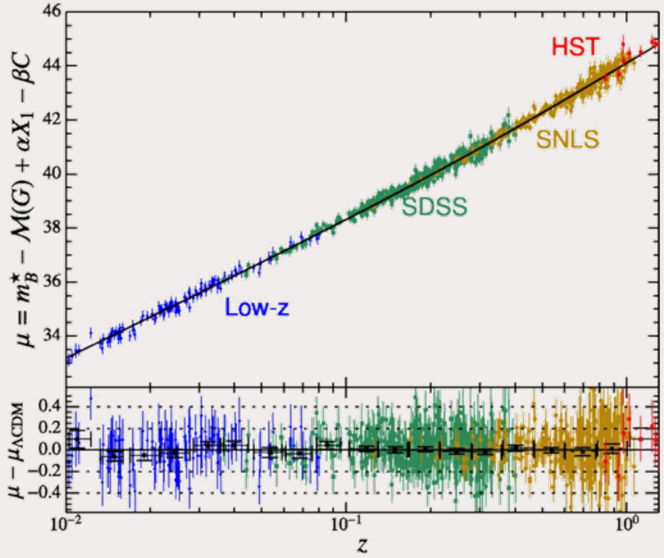
The Big-Bang Universe

The Universe is expanding

$$v = Hd$$



E. Hubble (1929)

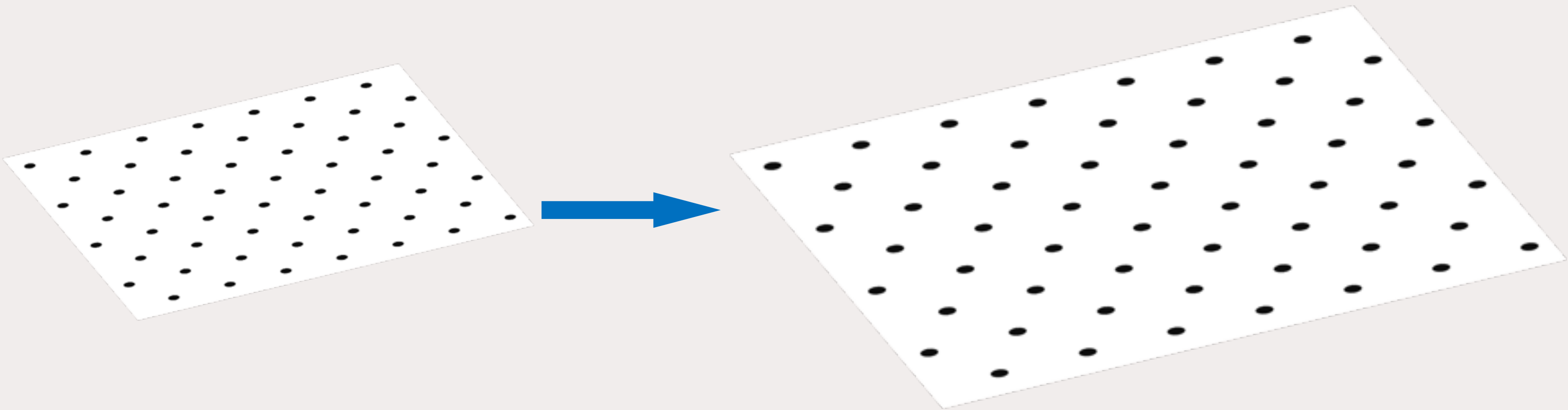


Betoule M, et al. Astron Astrophys 568 (2014) 22

The Big-Bang Universe

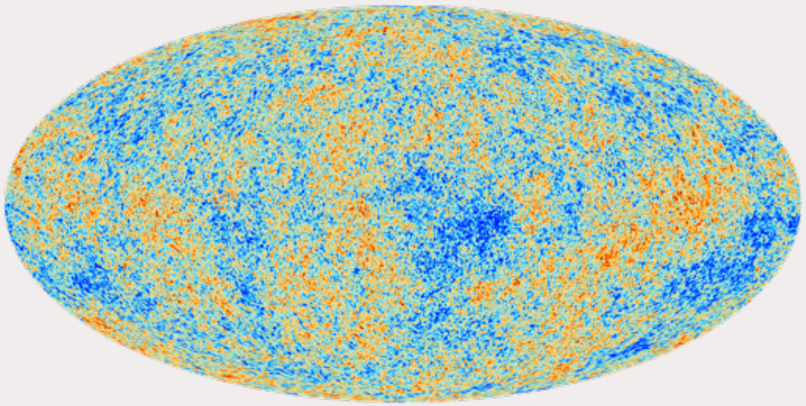
The Universe is expanding

$$v = Hd$$



$$r = a(t)x \qquad \dot{r} = \dot{a}x \qquad v = \dot{r} = \frac{\dot{a}}{a}r \qquad H = \frac{\dot{a}}{a}$$

2. The universe is homogeneous and isotropic at large scales.



Manifest in CMB sky

$$T \sim 2.7\mathrm{K}$$

$$\Delta T/T \sim 10^{-5}$$

Matter distribution is independent of position.

In comoving frame,

$$T_{\mu\nu} = \mathrm{diag}(\rho, p, p, p)$$

The Big-Bang Universe

Einstein's equation with homogeneous & isotropic & flat 3d slice -> Friedmann's equation

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$3M_{\rm Pl}^2 H^2 = \rho$$

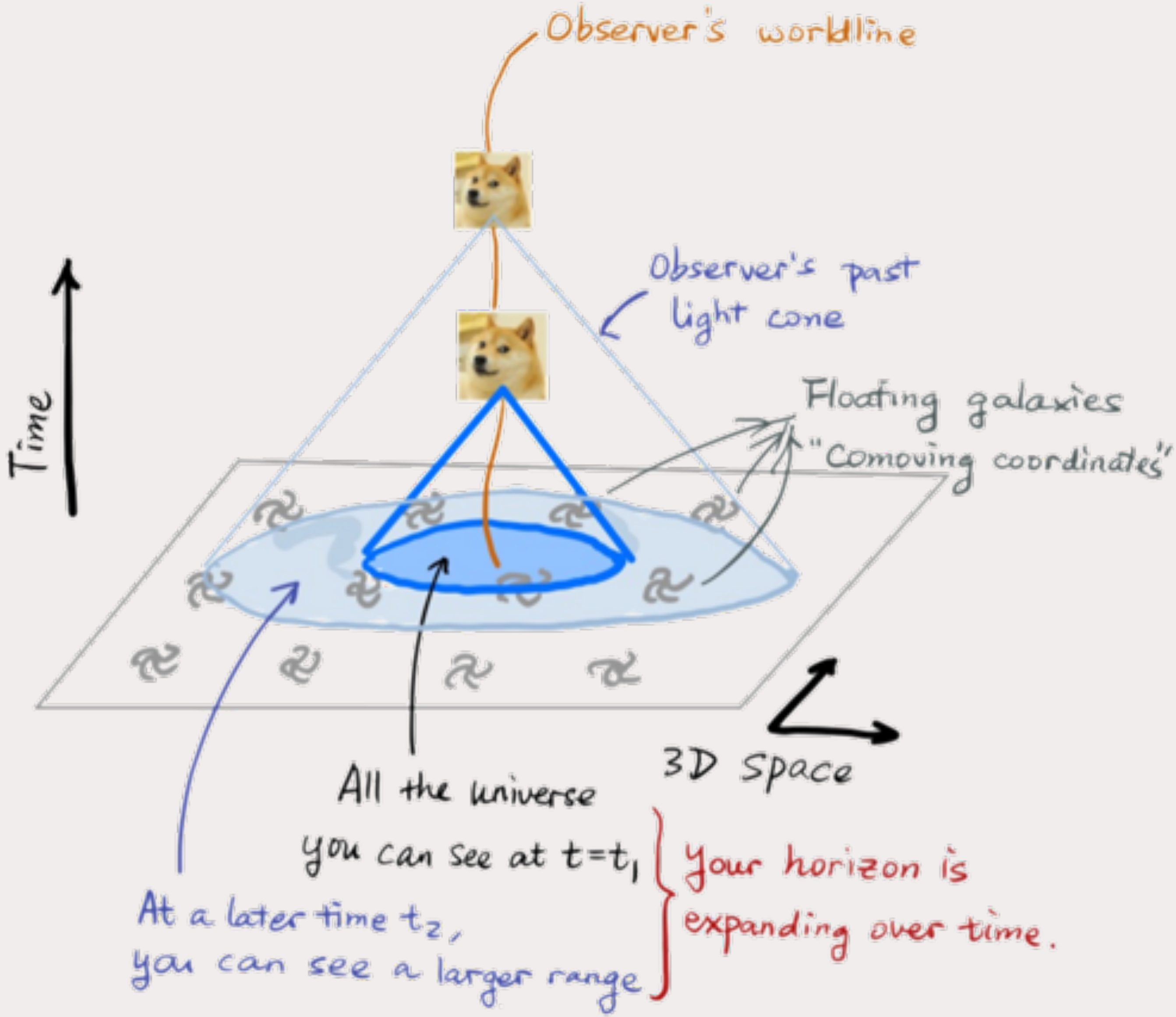
$$M_{\rm Pl} = (8\pi G)^{-1/2} \simeq 2.4 \times 10^{18}\,\mathrm{GeV}$$

$$p = p(\rho)$$

$$\dot\rho + 3H(p+\rho) = 0$$

Cold matter:	$p = 0$	$\rho \propto a^{-3}$	$a \propto t^{2/3}$	$H = \dfrac{2}{3t}$
Radiation:	$p = \rho/3$	$\rho \propto a^{-4}$	$a \propto t^{1/2}$	$H = \dfrac{1}{2t}$
Cosmo. const.	$p = -\rho$	$\rho = \text{const.}$	$a \propto e^{Ht}$	$H = \text{const.}$

Horizon


Observer's worldline
Observer's past light cone
Floating galaxies
"Comoving coordinates"
Time
3D Space
All the universe you can see at t=t₁
At a later time t₂, you can see a larger range
Your horizon is expanding over time.

Horizon

The range of universe you can see is your past light cone, and is time-dependent. It is the total distance traveled by light from an initial time t_i to the moment of observation t_f.

In flat space this is simply t_f - t_i .

In expanding universe, the comoving size of the horizon is

$$\tau = \int_{t_i}^{t_f} \frac{\mathrm{d}t}{a(t)}$$

In terms of τ, the metric becomes flat up to a "conformal factor," so τ is also called the "conformal time." But it is really the comoving size of the horizon.

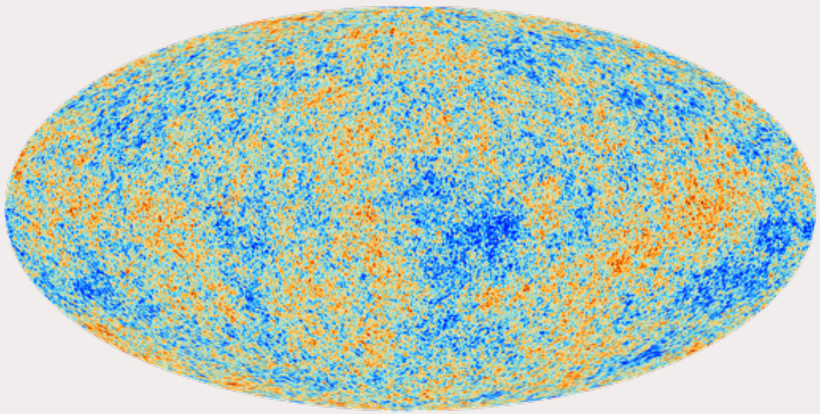
Horizon

$$\tau = \int_{t_i}^{t_f} \frac{\mathrm{d}t}{a(t)}$$

In a universe expanding "slowly enough," the previous cartoon still holds: the horizon increases with time.

Cold matter:	$a \propto t^{2/3}$	$\tau \propto t^{1/3}$	Expanding
Radiation:	$a \propto t^{1/2}$	$\tau \propto t^{1/2}$	Expanding
Cosmo. const.	$a \propto e^{Ht}$	$\tau \propto e^{-Ht}$	Contracting

Fluctuations



Fluctuations are small:

$$\delta T/\bar{T} \sim 10^{-5}$$

=> Linear perturbation theory works well at leading order

=> k-modes evolve independently at leading order

$$\phi(\mathbf{x}) = \bar{\phi} + \delta\phi(\mathbf{x}) \qquad \delta\phi(\mathbf{x}) = \int \mathrm{d}^3\mathbf{x}\, e^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}}\delta\phi_{\mathbf{k}}$$

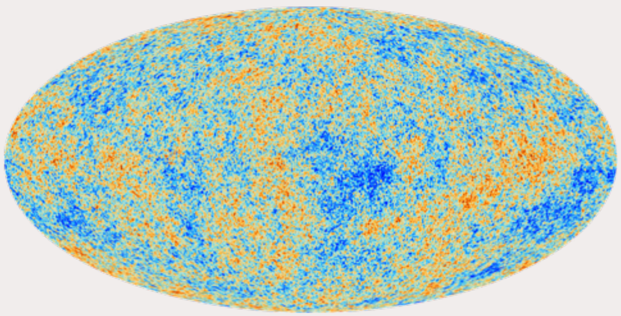
How to collect information from 2D / 3D fluctuation maps?

$$\langle \delta\phi_{\mathbf{k}_1}\delta\phi_{\mathbf{k}_2}\cdots\delta\phi_{\mathbf{k}_n}\rangle$$

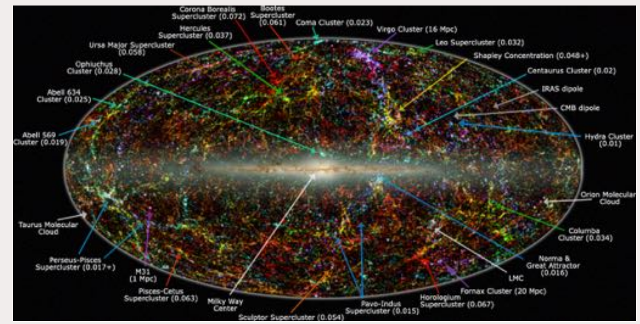
Fluctuations

Example: 2-point correlator (power spectrum)

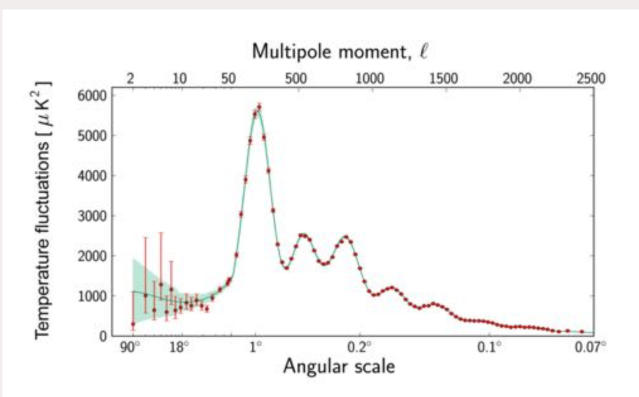
Photon

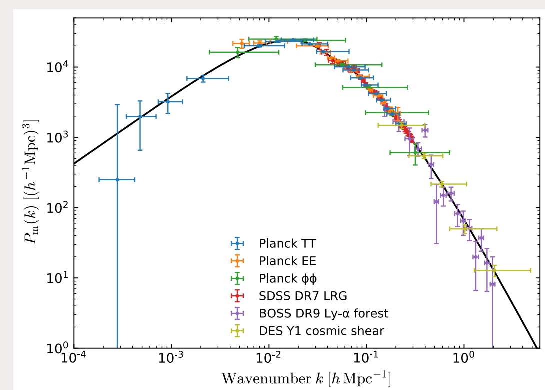


Matter



arXiv:astro-ph/0405069





1807.06205

Tracing the evolution of fluctuations

Fluctuations
at very early time t_0

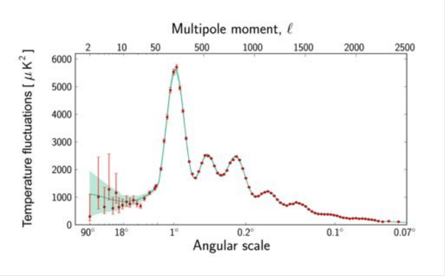
$\delta T(\mathbf{k}, t_0)$

?

Einstein equation
+ Boltzmann equation

Observable today

$\delta T(\mathbf{k}, t)$

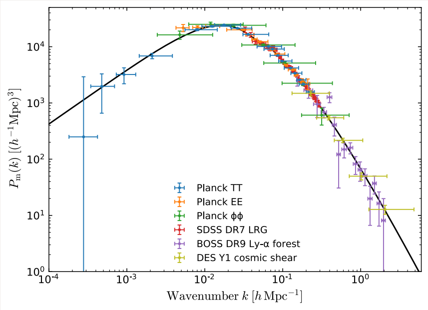


$\delta \rho(\mathbf{k}, t_0)$

?

Evolving the observed fluctuation backwards in time, we can get their distribution at very early time. These are mostly well-known physics.

$\delta \rho(\mathbf{k}, t)$



Tracing the evolution of fluctuations

Fluctuations
at very early time t_0

$\delta T(\mathbf{k}, t_0)$

?

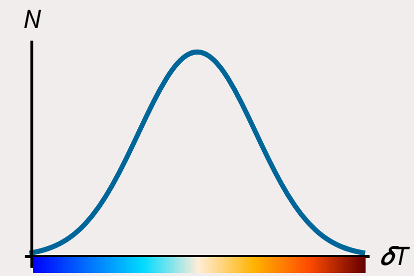
$\delta \rho(\mathbf{k}, t_0)$

?

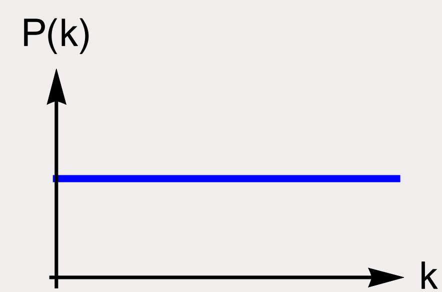
$$\zeta(\mathbf{k}, t_0) \sim 10^{-5}$$

“curvature fluctuation”

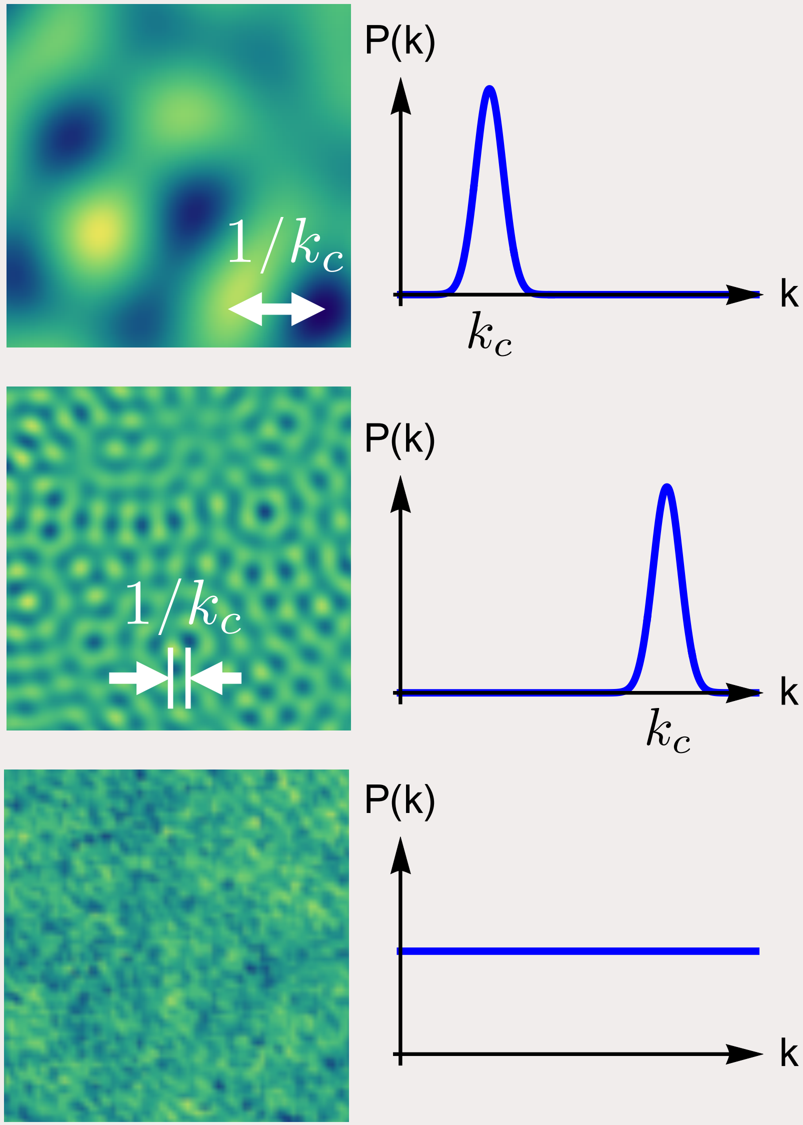
Most of fluctuations can be traced back to a common origin, the fluctuations in the scale factor, called the “curvature fluctuation”
[One exception: primordial GW]



The distribution of curvature fluctuations is pretty uniform, Gaussian, and scale invariant.



What is scale invariance?



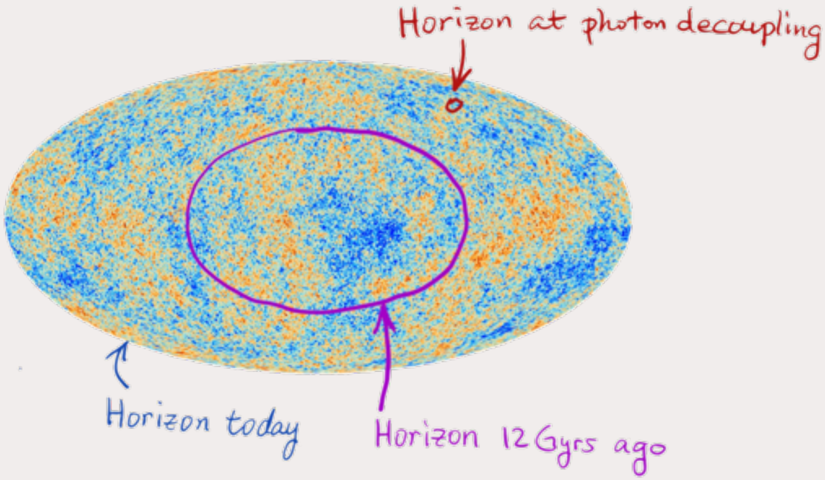
Scale invariance: the amount of fluactuation per k-bin is invariant under scaling

Exercise Draw a schematic power spectrum of this picture

The puzzle

In most of cosmic history, the horizon has been expanding

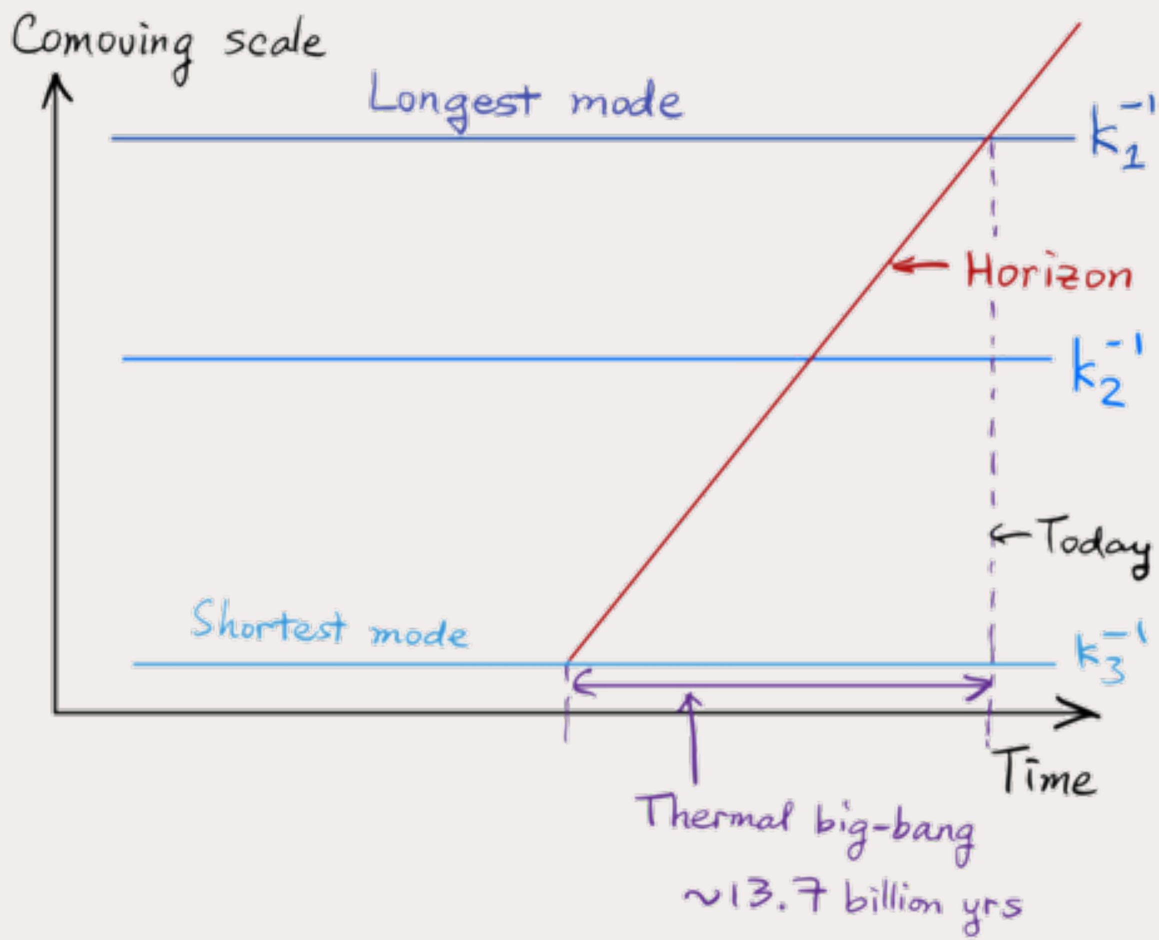
Fluctuations were outside the horizon early on.



Out of many causally independent patches, a rather uniform, scale-invariant, and Gaussian distribution of fluctuations was built.

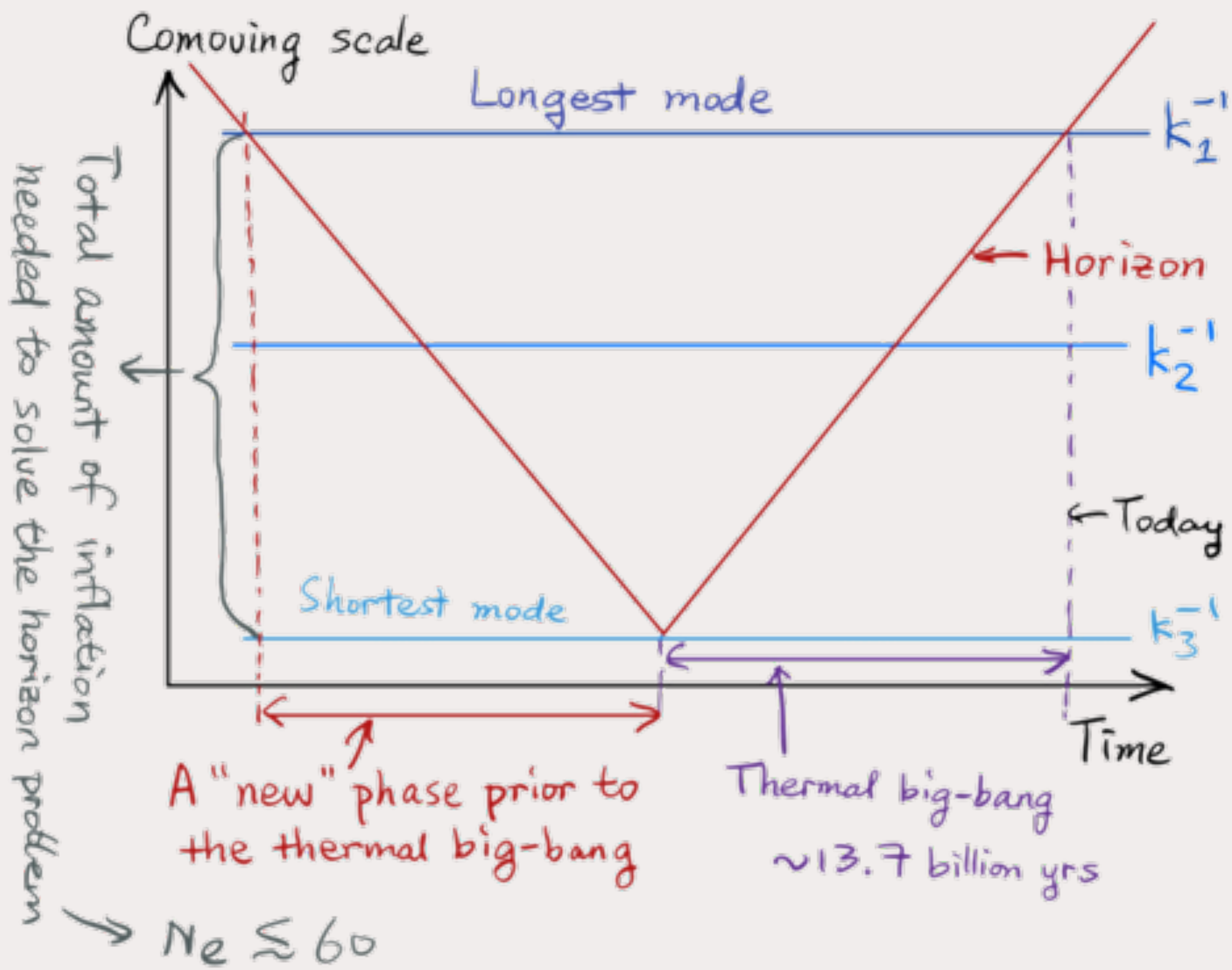
Why?

The puzzle



Comoving scale
Longest mode
k_1^{-1}
Horizon
k_2^{-1}
Today
Shortest mode
k_3^{-1}
Time
Thermal big-bang
~13.7 billion yrs

A solution


Comoving scale
Longest mode
k_1^{-1}
Horizon
k_2^{-1}
Today
Shortest mode
k_3^{-1}
Time
Total amount of inflation
needed to solve the horizon problem
→ $N_e \lesssim 60$
A "new" phase prior to
the thermal big-bang
Thermal big-bang
~13.7 billion yrs

Solution

Solution: a new phase of cosmic evolution with shrinking horizon, lasting ~ 60 e-folds or longer

Simplest and most elegant solution: expansion by cosmo. const. or something similar => inflation

A cosmological constant does not work by itself
=> Inflation forever. We need a mechanism to end the inflation and to initiate the thermal big bang.

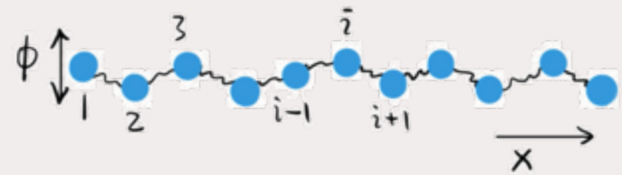
Guth, Linde, Starobinsky, 1970s

Solution

Identify the cc as the potential energy of a scalar field, called the inflaton

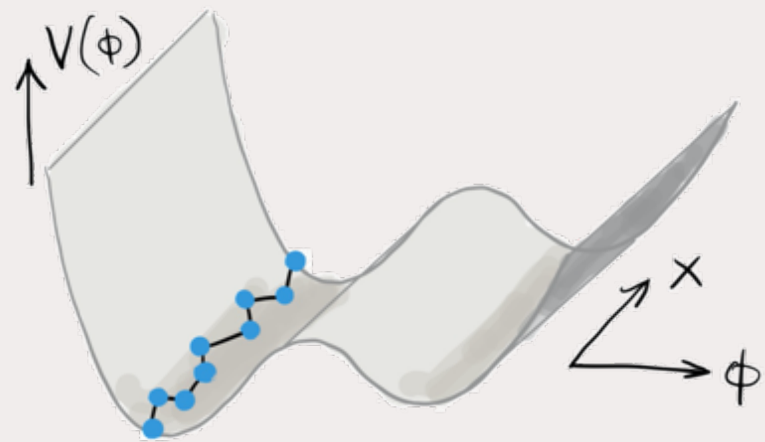
What is a field?

In particle physics each particle is associated with a field. Particles are identified as the excitations of corresponding fields.



$$E[\phi(x)] = \sum_i \left[\frac{1}{2}\dot{\phi}_i^2 + \frac{1}{2}\left(\frac{\phi_i - \phi_{i-1}}{\Delta x}\right)^2 \right]$$

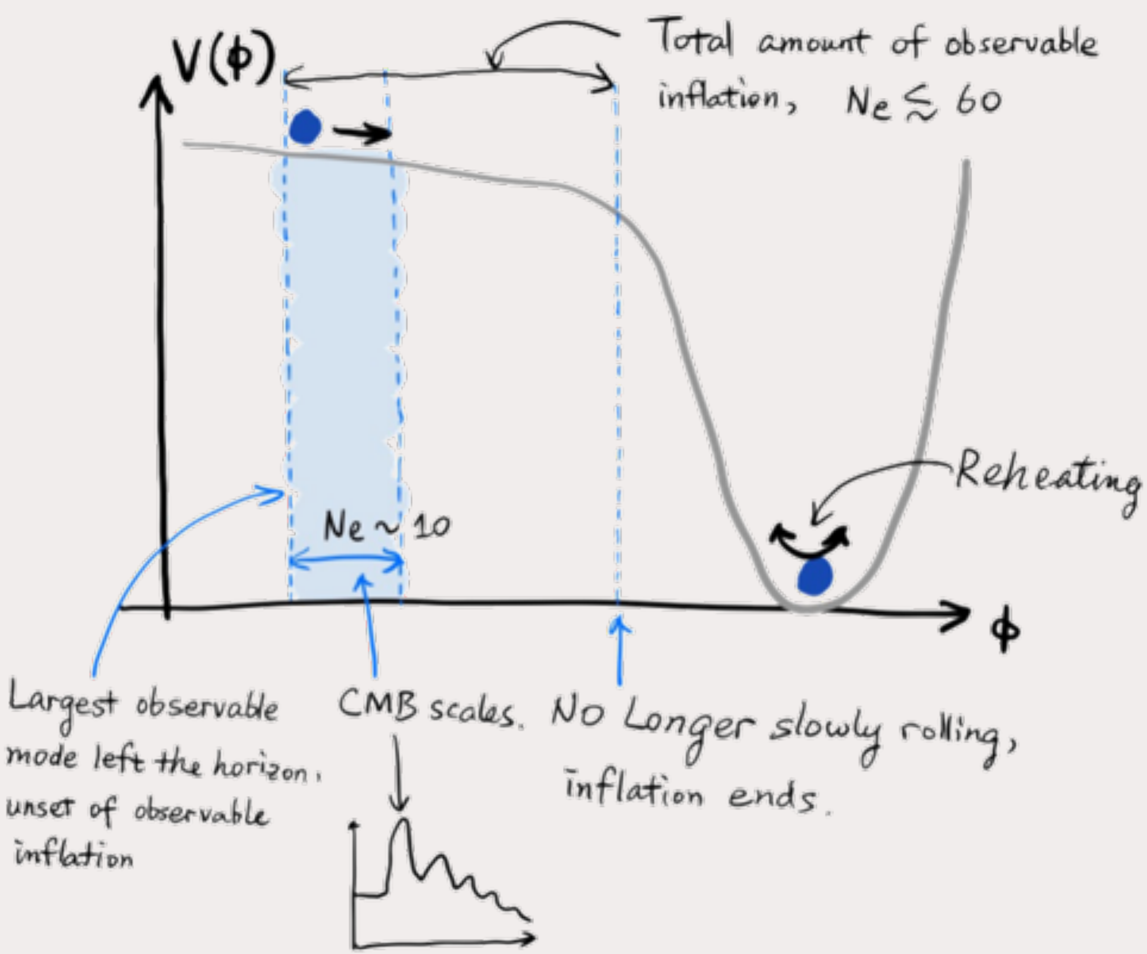
$$\Rightarrow \dot{\phi}^2(t,x) + \left[\frac{\partial \phi(t,x)}{\partial x}\right]^2$$



$$E[\phi(x)] = \dot{\phi}^2 + \left(\frac{\partial}{\partial x}\phi\right)^2 + V(\phi)$$

Slow-roll inflation

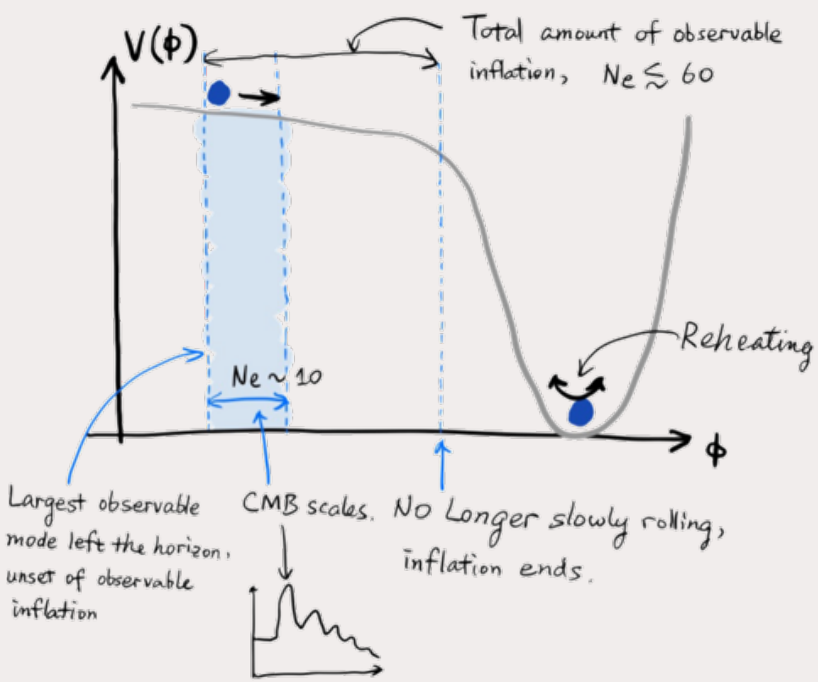
An inflaton rolling very slowly along its very potential



Slow-roll inflation

Challenge:

Any good model to realize the inflation quantitatively?



The height of inflaton potential is typically much higher than any energy scales we have probed.

Model realizations need physics at very high energies.

More fascinatingly, from inflation observables we could learn physics at high scales. Possibly the highest energy we could ever probe in nature.

What does an inflation model have to do?

At background level: providing enough (>~60 e-folds) accelerating expansion, and a mechanism to end the inflation.

Perturbation level: generating curvature fluctuations $\sim 10^{-5}$ consistent with observations.

Background level

Inflation potential $V(\phi)$ drives the expansion.

$$\ddot{\phi} + 3H\dot{\phi} + V'_\phi = 0 \qquad 3M_{\rm Pl}^2 H^2 = \tfrac{1}{2}\dot{\phi}^2 + V(\phi)$$

Energy and pressure $\rho = \dfrac{1}{2}\dot{\phi}^2 + V(\phi)$ $\quad p = \dfrac{1}{2}\dot{\phi}^2 - V(\phi)$

We want the inflaton potential to be like a cc, $p = -\rho$, this requires

$$\tfrac{1}{2}\dot{\phi}^2 \ll V(\phi)$$

We also want the inflaton to roll slowly enough to have enough # e-folds, so

$$\ddot{\phi} \ll H\dot{\phi}$$

Background level

Slow roll conditions: $\frac{1}{2}\dot{\phi}^2 \ll V(\phi)$ $\ddot{\phi} \ll H\dot{\phi}$

In literature the slow-roll parameters are often used:

$$\epsilon = -\frac{\dot{H}}{H^2} \qquad \eta = \frac{\dot{\epsilon}}{H\epsilon}$$

Alternative definitions in terms of potential alone,

$$\epsilon_V = \frac{1}{2}M_{\rm Pl}^2(V'_\phi/V)^2 \qquad \eta_V = M_{\rm Pl}^2 V''_\phi/V$$

$$\epsilon \simeq \epsilon_V \qquad \eta \simeq 4\epsilon_V - 2\eta_V$$

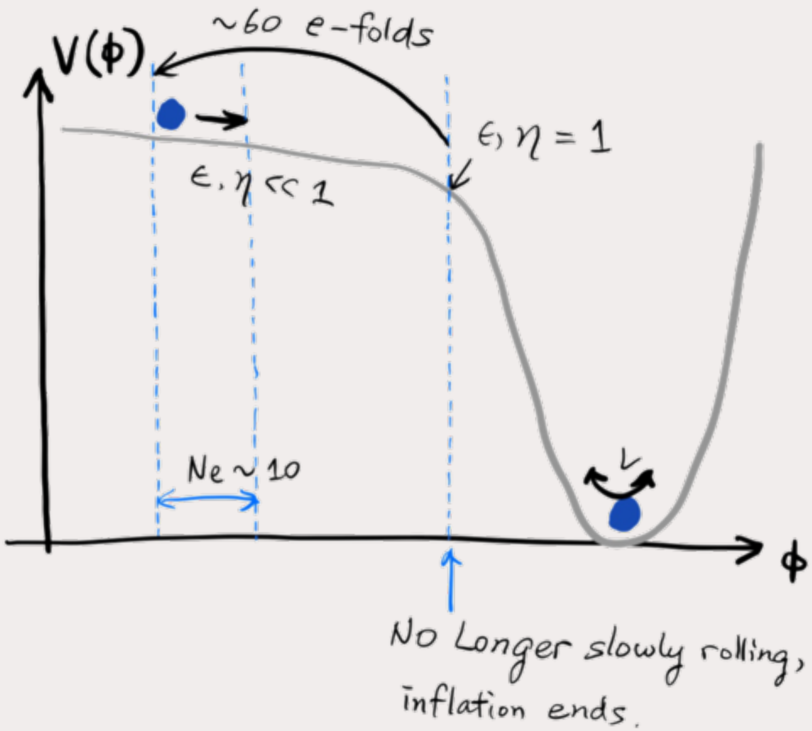
Inflation requires $\epsilon \ll 1, \eta \ll 1$

Background level

Inflation requires $\epsilon \ll 1, \eta \ll 1$

Whenever this condition fails to hold, the inflation ends.

Practically, easier to find the point in field space where slow roll parameters first reach 1. Then we evolve the field backwards for ~60 e-folds, to find the "initial" position of the field at the beginning of observable inflation.



Perturbations

The background evolution is classical

Quantum fluctuations on top of classical evolution generates the curvature perturbation.

$$\phi(t,\mathbf{x}) = \phi_0(t) + \varphi(t,\mathbf{x}) \quad \Rightarrow \quad \varphi_k(t)$$

$$\varphi_k'' - \frac{2}{\tau}\varphi_k' + k^2\varphi_k = 0$$

Easier to use conformal time τ. Mass can be neglected.

$$t \in (-\infty, +\infty)$$

$$\tau \in (-\infty, 0)$$

Perturbations

$$\varphi_k = \frac{H}{\sqrt{2k^3}}(1 + \mathrm{i}k\tau)e^{-\mathrm{i}k\tau}$$

Initial condition: modes were well within horizon at early times, like in Minkowski vacuum.

Fluctuations are frozen to a const at late times, and seed the fluctuations outside the horizon as we see today.

$$\varphi_k \to \frac{H}{\sqrt{2k^3}}$$

Fluctuations in inflaton induces fluctuations in curvature (fluctuations in the scale factor)

$$\zeta_k = -\frac{H}{\dot{\phi}_0}\varphi_k \qquad \langle \zeta_k^2 \rangle = \frac{H^4}{\dot{\phi}_0^2}\frac{1}{2k^3} \equiv \frac{2\pi^2}{k^3}P_\zeta(k)$$

$$P_\zeta(k) = \frac{H^4}{4\pi^3\dot{\phi}_0^2} = \frac{H^2}{8\pi^2\epsilon M_{\mathrm{Pl}}^2} \simeq 2 \times 10^{-9}.$$

Perturbations

$$P_\zeta(k) = \frac{H^4}{4\pi^3 \dot{\phi}_0^2} = \frac{H^2}{8\pi^2 \epsilon M_{\rm Pl}^2} \simeq 2 \times 10^{-9}.$$

In addition to the scale factor, the spatial metric can also fluctuate => primordial gravitational waves.

$$P_g(k) = \frac{2H^2}{\pi^2 M_{\rm Pl}^2} \qquad\qquad r \equiv P_g/P_\zeta = 16\epsilon$$

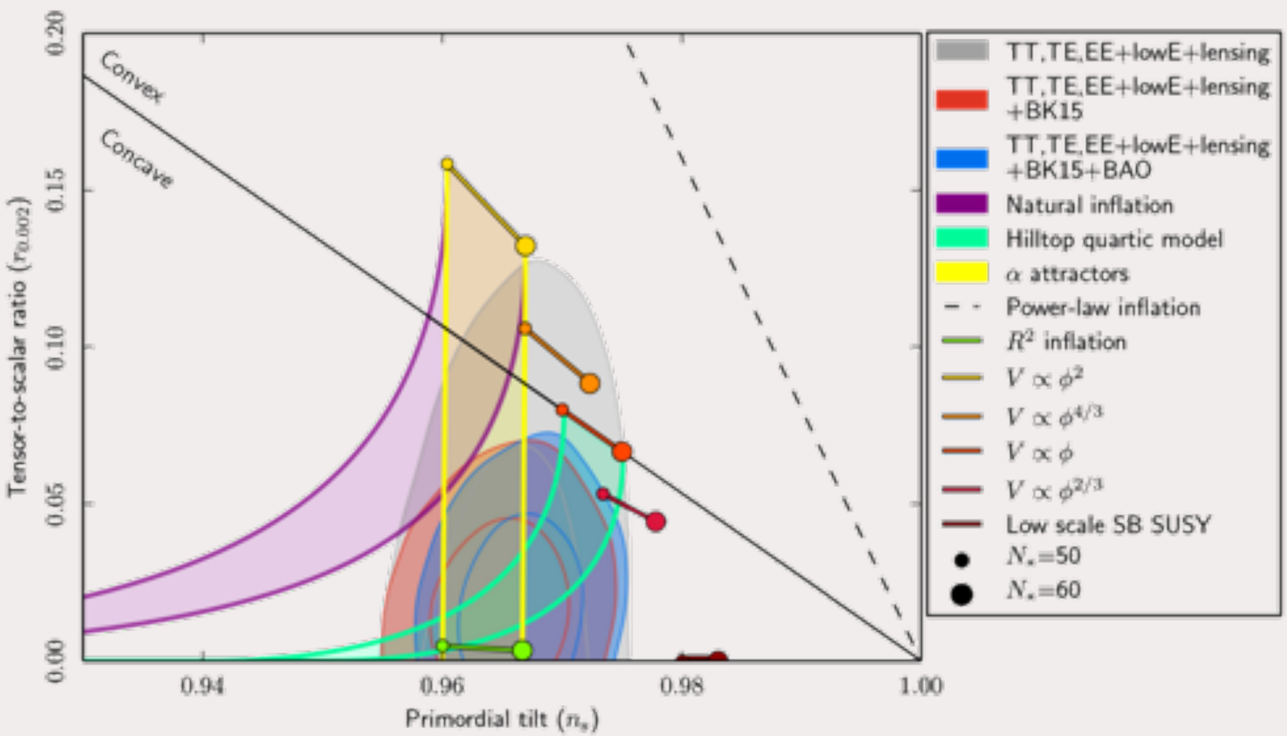
Scale invariance, but not exactly: inflaton is rolling, H is slowly decreasing, too => scalar power spectrum decreases slightly with k

$$\frac{{\rm d}\log P_\zeta(k)}{{\rm d}\log k} = -2\epsilon - \eta \equiv n_s - 1$$

Models vs data

$$r \equiv P_g/P_\zeta = 16\epsilon$$

$$n_s = 1 - 2\epsilon - \eta$$



Planck 2018

Building an inflation model

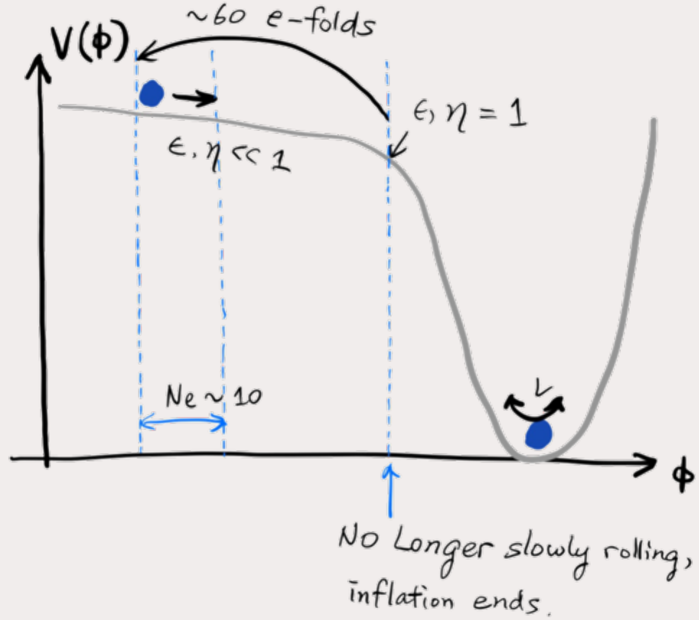
At the level of 2-point correlations, all we have measured are

$$P_\zeta \simeq \frac{H^2}{8\pi^2\epsilon M_{\rm Pl}^2} \simeq 2\times 10^{-9}$$

$$n_s = 1 - 2\epsilon - \eta \simeq 0.965$$

$$r = 16\epsilon \lesssim 0.056$$

The problem then is to build an inflation model consistent with these observations at CMB scale.

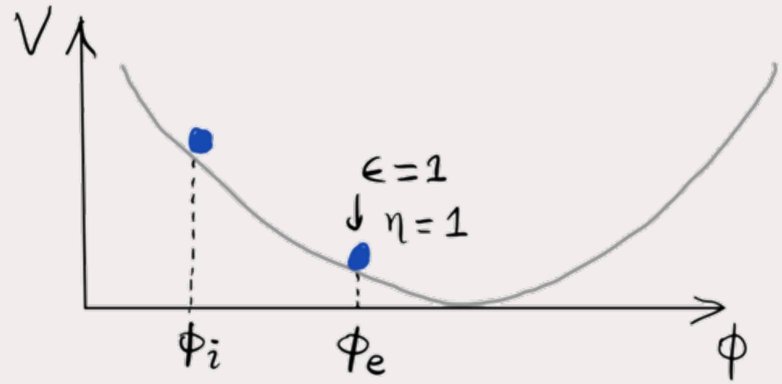


If we were allowed to draw the shape of V(ϕ) at will, then super easy. But much more constrained when including various particle physics input

An example of quadratic potential

$$V(\phi) = \tfrac{1}{2} m^2 \phi^2 \qquad \epsilon_V = \tfrac{1}{2} M_{\rm Pl}^2 (V'_\phi / V)^2 \qquad \eta_V = M_{\rm Pl}^2 V''_\phi / V$$

$$\epsilon_V(\phi) = \eta_V(\phi) = 2(M_{\rm Pl}/\phi)^2 \qquad \Rightarrow \qquad \phi_e = \sqrt{2} M_{\rm Pl}$$



$$N_e = \int_{t_i}^{t_e} H \mathrm{d}t = \int_{\phi_i}^{\phi_e} \frac{H}{\dot\phi} \mathrm{d}\phi \simeq \frac{-1}{M_{\rm Pl}} \int_{\phi_i}^{\phi_e} \frac{\mathrm{d}\phi}{\sqrt{2\epsilon}} = \frac{\phi_i^2 - \phi_e^2}{4M_{\rm Pl}^2} \simeq \frac{\phi_i^2}{4M_{\rm Pl}^2}$$

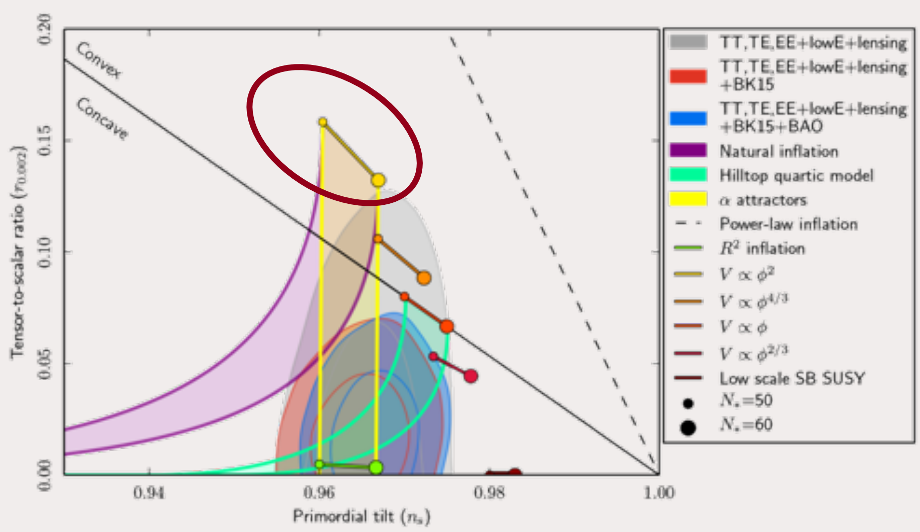
$$\phi_i \simeq 2\sqrt{N_e} M_{\rm Pl} \qquad \epsilon_V(\phi_i) = \eta_V(\phi_i) \simeq \frac{1}{2N_e}$$

An example of quadratic potential

$$r \simeq 16\epsilon_V \simeq \frac{8}{N_e} \qquad n_s \simeq 1 - 6\epsilon_V + 2\eta_V \simeq 1 - \frac{2}{N_e}$$

$$A_s = \frac{H^2}{8\pi^2 \epsilon M_{\rm Pl}^2} = 2 \times 10^{-9} \quad \Rightarrow \quad H \sim 10^{14}\,{\rm GeV}$$

$$\rho \sim 10^{16}\,{\rm GeV} \qquad m \sim 10^{13}\,{\rm GeV} \qquad \phi_i \simeq 2\sqrt{N_e} M_{\rm Pl} \sim \mathcal{O}(10) M_{\rm Pl}$$

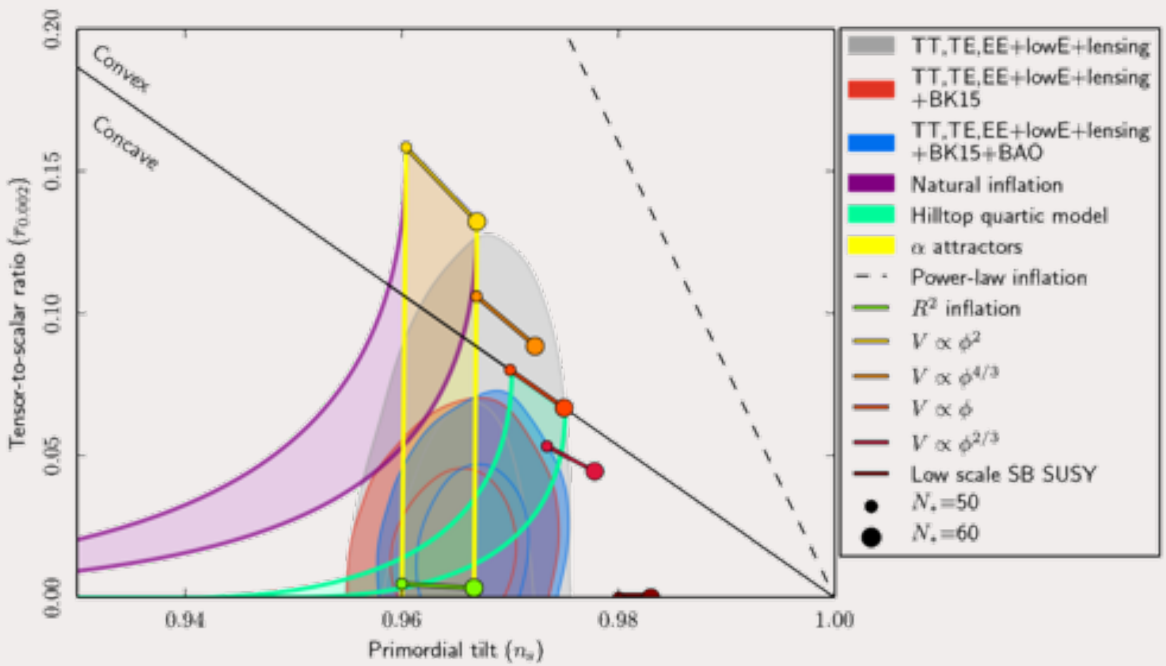


More examples

General power law $V \sim |\phi|^p$ (p=2/3: monodromy inflation)

Exercise: derive inflation observables for general *p*

Starobinsky model / Higgs inflation $V \sim 1 - \exp(-\alpha\phi)$

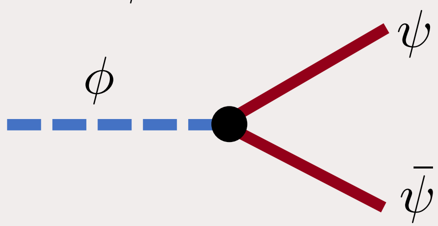


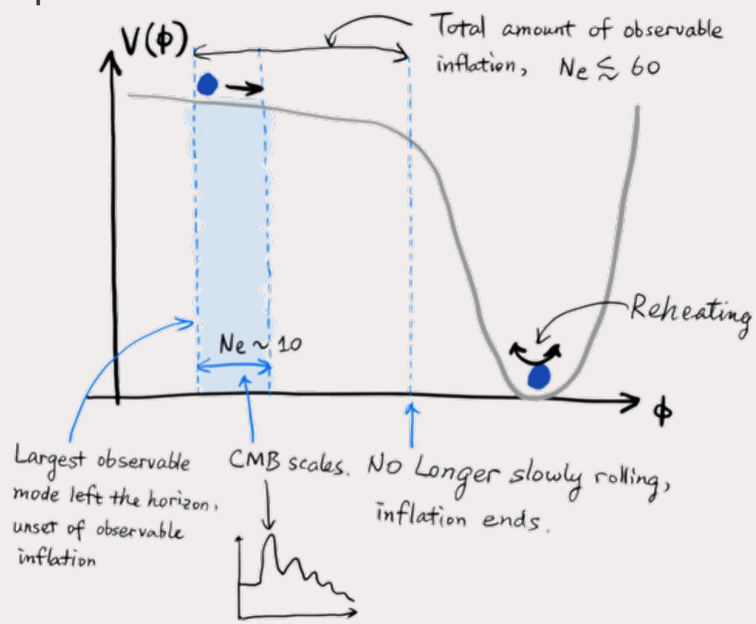
Reheating

After the inflation, the inflaton oscillates around the bottom of the potential, and decays to other particles.

Example: $y\phi\bar{\psi}\psi$

$\Gamma \sim y^2 m_\phi \qquad H \propto t^{-1} \qquad H \sim \Gamma$





$$\rho \sim M_{\rm Pl}^2 H^2 \sim y^4 M_{\rm Pl}^2 m_\phi^2$$

$$T < \rho^{1/4} \sim y\sqrt{M_{\rm Pl} m_\phi}$$

When inflaton decays into bosons with large couplings, nonperturbative effects can be important (preheating)

For a good review, see Lozanov: 1907.04402

General Remarks

In the (simplest) single-field slow-roll:

$$\left.\begin{aligned} A_s \simeq \frac{\rho}{24\pi^2\epsilon M_{\rm Pl}^4} \simeq 2\times 10^{-9} \\ r \simeq 16\epsilon \end{aligned}\right\} \quad \Rightarrow \quad \frac{r}{0.05} \simeq \left(\frac{E}{1.5\times 10^{16}\mathrm{GeV}}\right)^4$$

The size of tensor mode ⇔ inflation scale

Non-observation of tensor modes sets an upper bound of inflation scale (at least in simple models):

E ~ < 10^{16} GeV,   H ~ < 10^{14} GeV

What is the lowest possible scale?

A hard lower bound: BBN, T > ~ 10MeV.

Exercise: What’s the corresponding Hubble scale?

General Remarks

The rolling distance is also related to the inflation scale

$$N_e \simeq -\frac{1}{M_{\text{Pl}}}\int_{\phi_i}^{\phi_e}\frac{\mathrm{d}\phi}{\sqrt{2\epsilon}} \qquad\qquad \Delta\phi \simeq \sqrt{2\epsilon}N_e M_{\text{Pl}}$$

High scale inflation ⇔ super-Planckian rolling. Swampland?

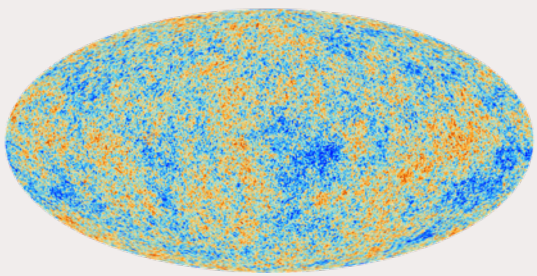
Low scale inflation needs extremely small ε (slope). Tuned?

Inflation model building is still challenging.

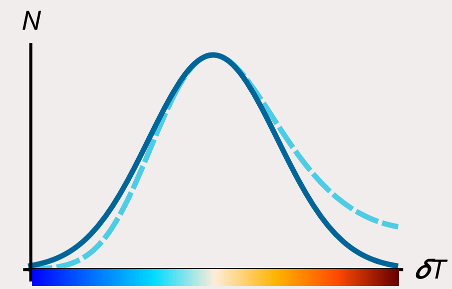
Alternatives: curvaton / modulated reheating / warm inflation …

Alternative to inflation. (slow) contraction / big bounce. Needs physics beyond GR + normal matter

Non-Gaussianity

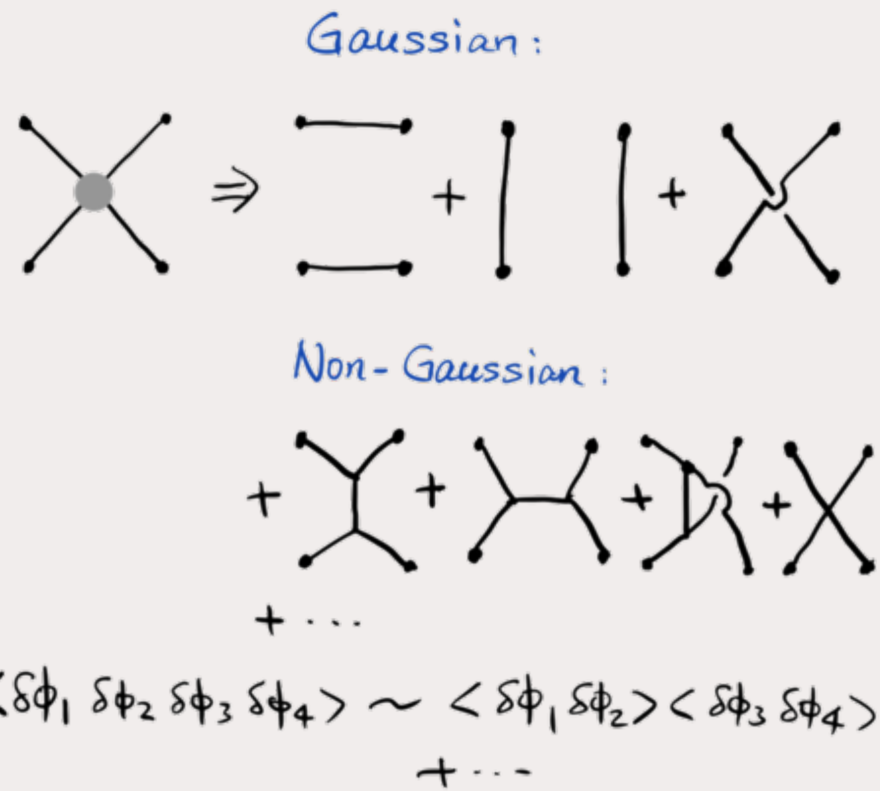


$$\langle \delta\phi_{\mathbf{k}_1}\delta\phi_{\mathbf{k}_2}\cdots\delta\phi_{\mathbf{k}_n}\rangle$$

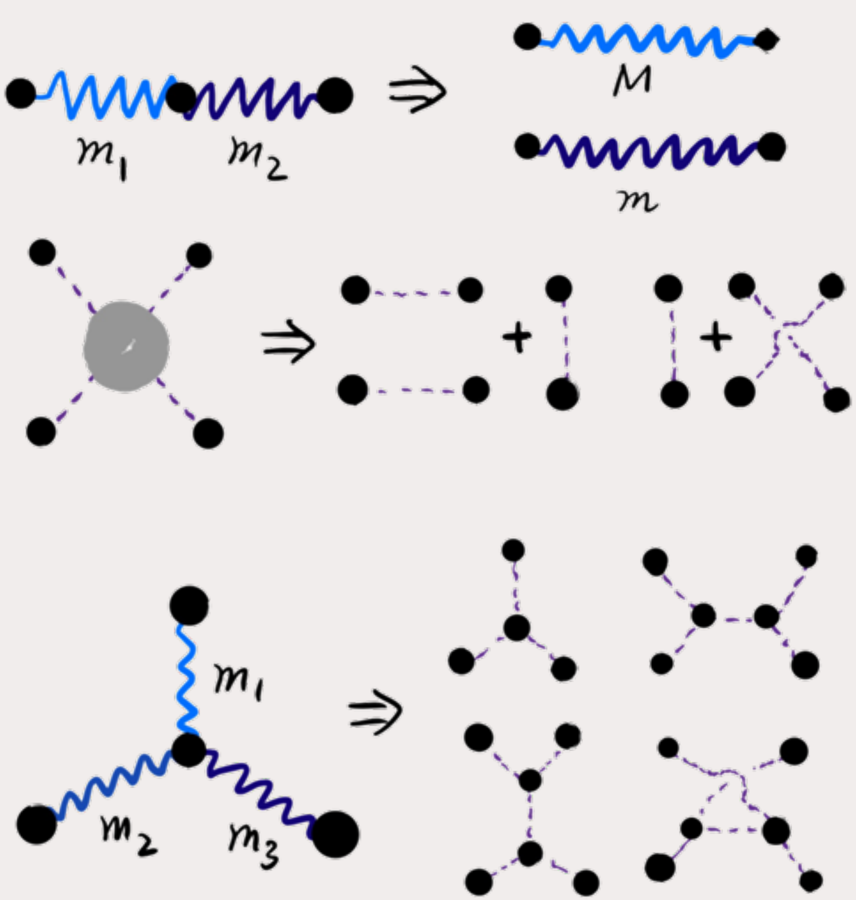


The observed primordial fluctuations are quite Gaussian, but there could be small non-Gaussianity.

Statistically, non-G means irreducible higher point correlations.



Non-Gaussianity



Physically, non-G can be nicely interpreted as the interactions of modes.

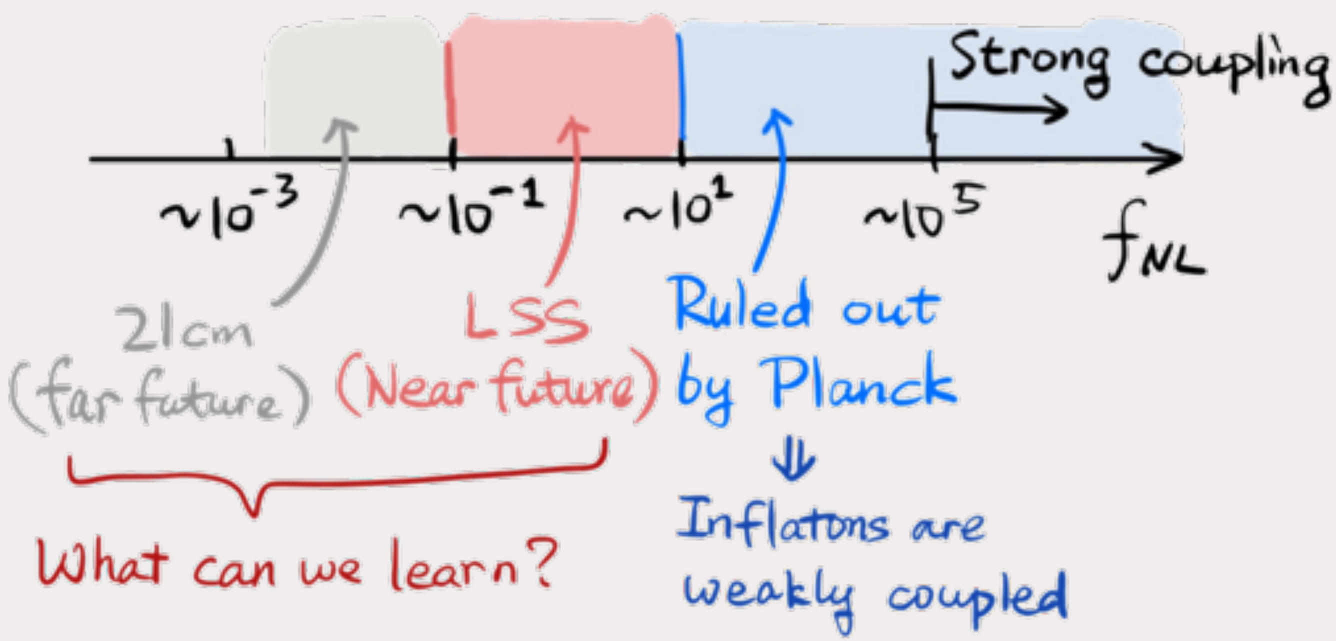
Free modes => Gaussian

But modes are not free. They are interacting, at least through gravity.
=> Non-Gaussianity

Non-Gaussianity

$$\langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\rangle = (2\pi)^7\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)P_\zeta^2\frac{1}{(k_1k_2k_3)^3}S(k_1,k_2,k_3)$$

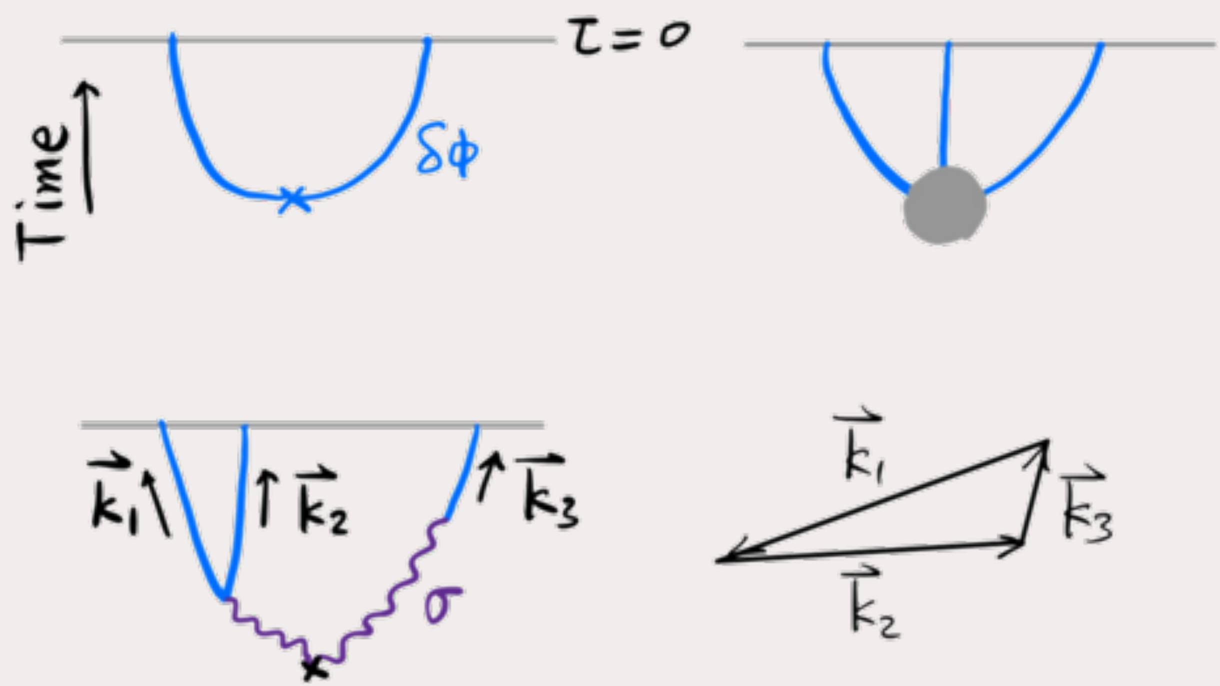
$$f_{\rm NL}\sim |S(k_1,k_2,k_3)|$$



Cosmological collider physics

Learning interactions at inflation scale

Especially, the production of a heavy particle



Production of a heavy particle

$$\sigma_k'' - \frac{2}{\tau}\sigma_k' + \left(k^2 + \frac{m^2}{H^2\tau^2}\right)\sigma_k = 0$$

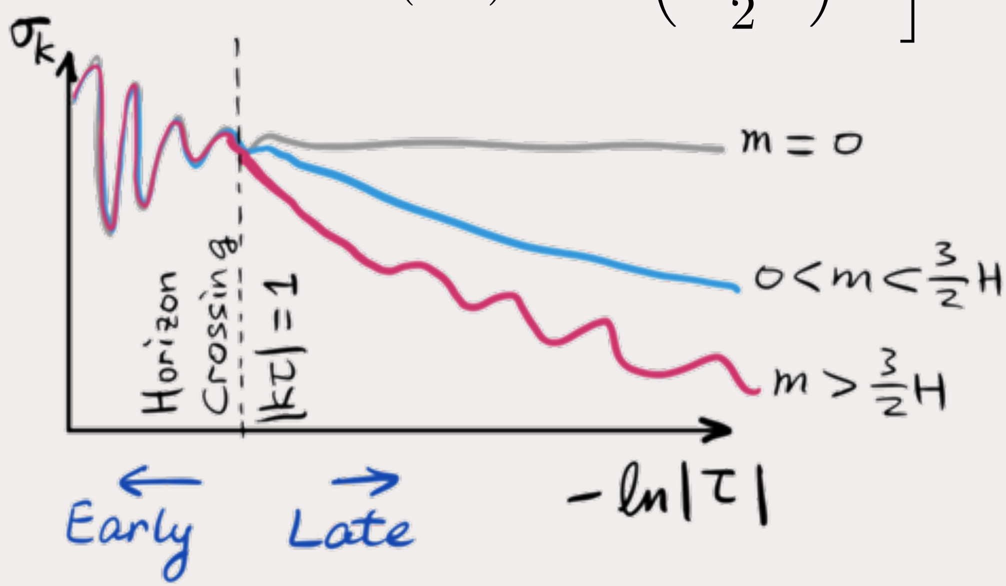
$\tau \to -\infty$

$$\sigma_k(\tau) \to \frac{\mathrm{i}H\tau}{\sqrt{2k}} e^{\mathrm{i}k\tau}$$

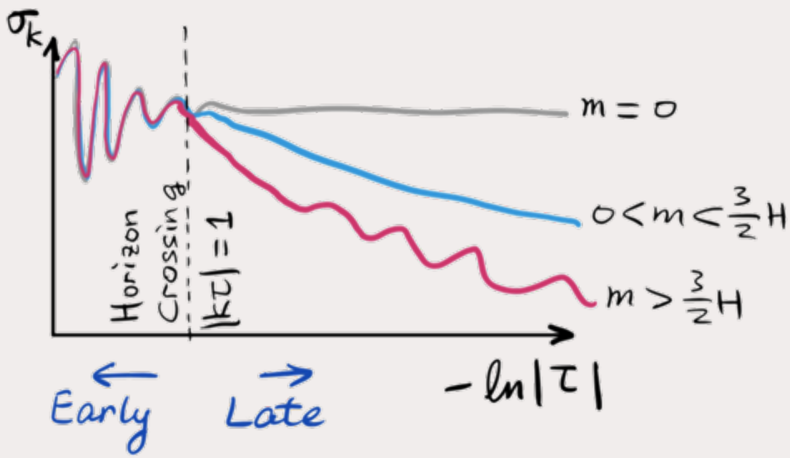
$\tau \to 0$

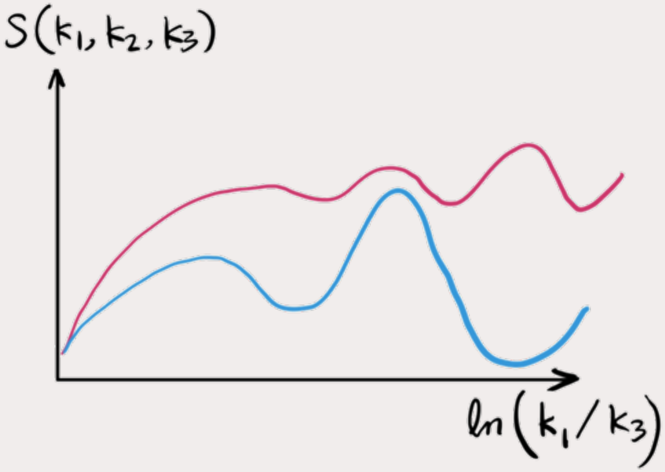
$$\sigma_k(\tau) \to \frac{H}{2\sqrt{\pi}}(-\tau)^{3/2}\left[\Gamma(-\nu)e^{-\mathrm{i}\nu/2}\left(\frac{-k\tau}{2}\right)^{\nu} + \Gamma(+\nu)e^{+\mathrm{i}\nu/2}\left(\frac{-k\tau}{2}\right)^{-\nu}\right]$$

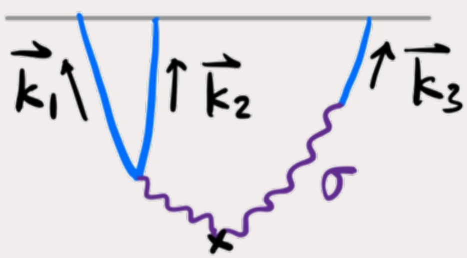
$$\nu \equiv \sqrt{\frac{9}{4} - \frac{m^2}{H^2}}$$

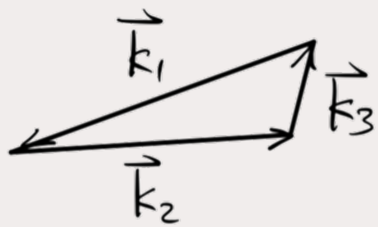


A massive particle in inflation







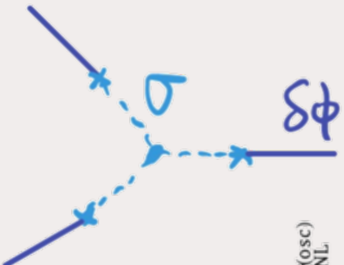


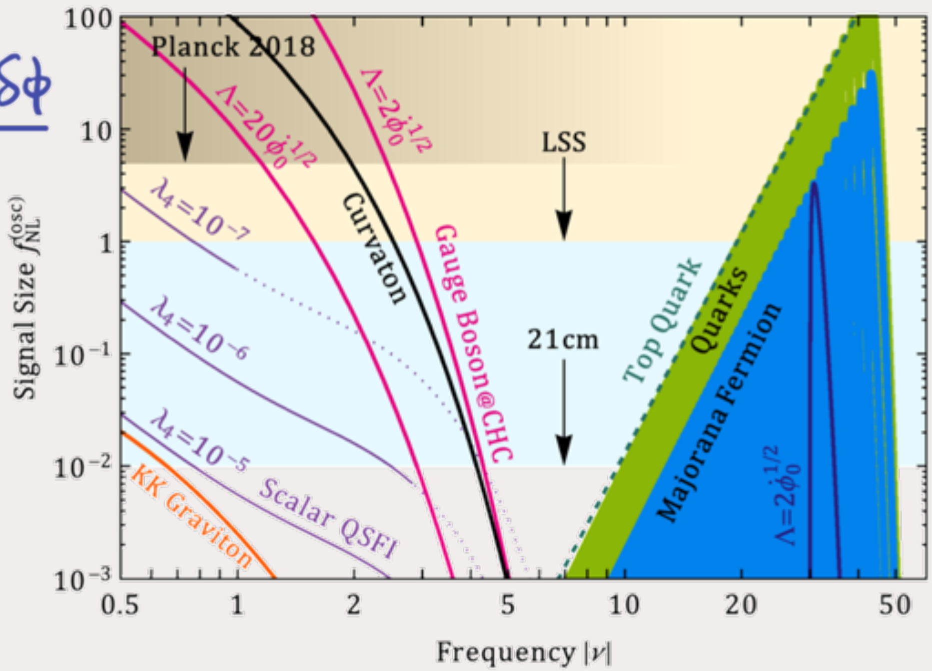
$$\int \mathrm{d}\tau \cdots (k_3\tau)^{\pm\nu} e^{\mathrm{i}(k_1+k_2)\tau} \sim (k_1/k_3)^{\pm\nu}$$

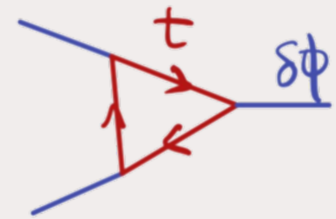
The frequency in $\ln(k_1/k_3)$ tells the mass

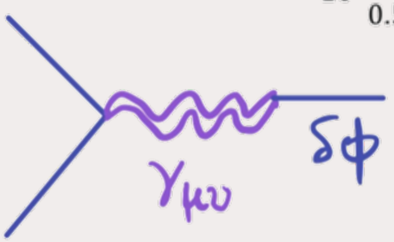
The angular dep. tells the spin


W
δh


σ
δϕ

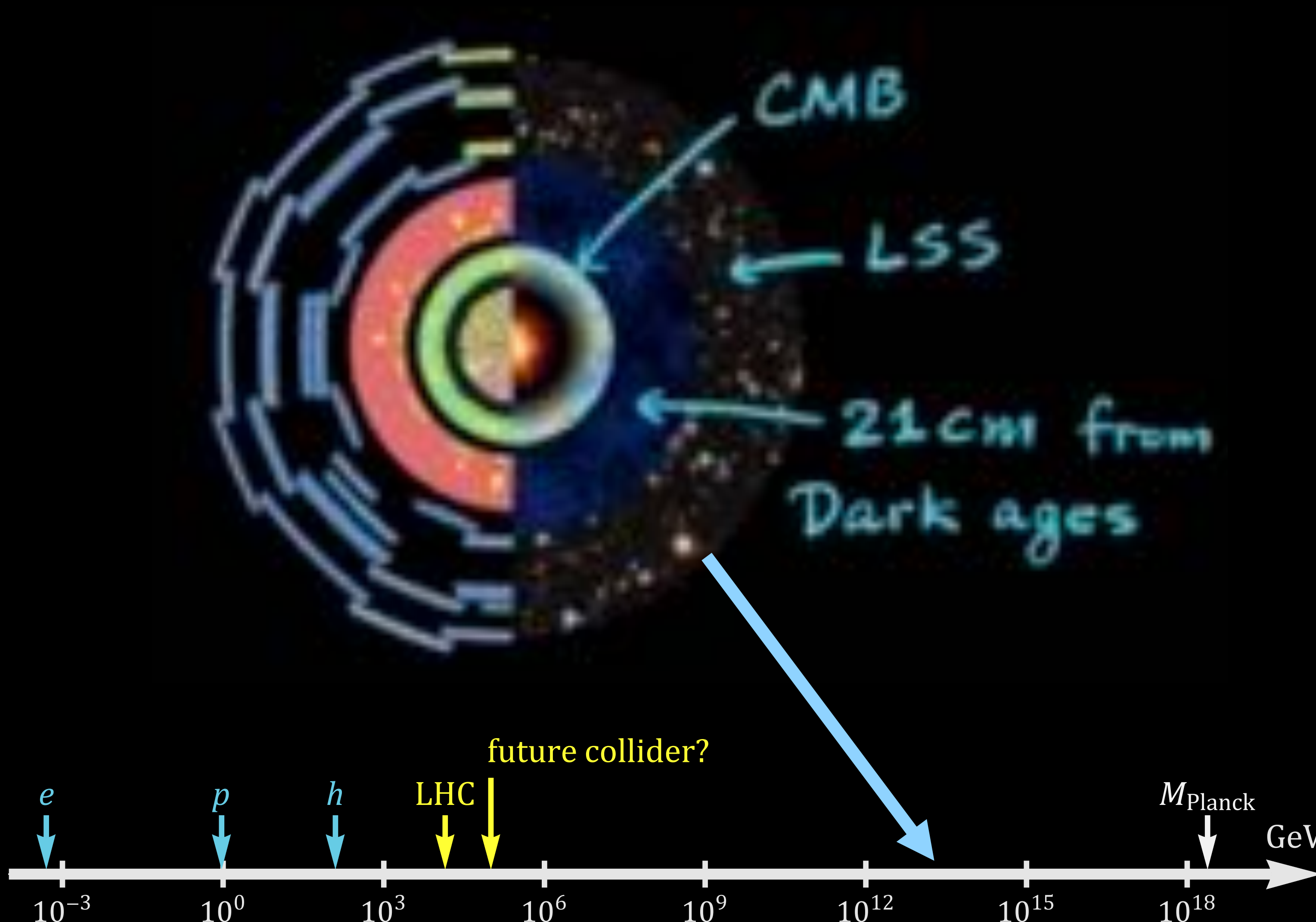

Planck 2018
Λ=20φ̇₀^{1/2}
Λ=2φ̇₀^{1/2}
Curvaton
Gauge Boson@CHC
LSS
21cm
Top Quark
Quarks
Majorana Fermion
Λ=2φ̇₀^{1/2}
λ4=10^-7
λ4=10^-6
λ4=10^-5
Scalar QSFI
KK Graviton
Signal Size f_NL^(osc)
100
10
1
10^-1
10^-2
10^-3
0.5
1
2
5
10
20
50
Frequency |ν|


t
δϕ


δϕ
γμν

1910.12876

Cosmological collider physics



A Summary

A period of fast expansion prior to the thermal big-bang solves the horizon problem

A simple realization: slow-roll inflation
Quantum fluctuations seed the density perturbations which eventually shape the large scale structure of our universe

Inflation theory has predictability
Scale invariant but not exactly so
But not easy to pin down the specific inflation model

Cosmological collider physics
Inflation provides us probably the highest energy accessible by realistic experiments / observations
A new and exciting way to study fundamental particle physics