

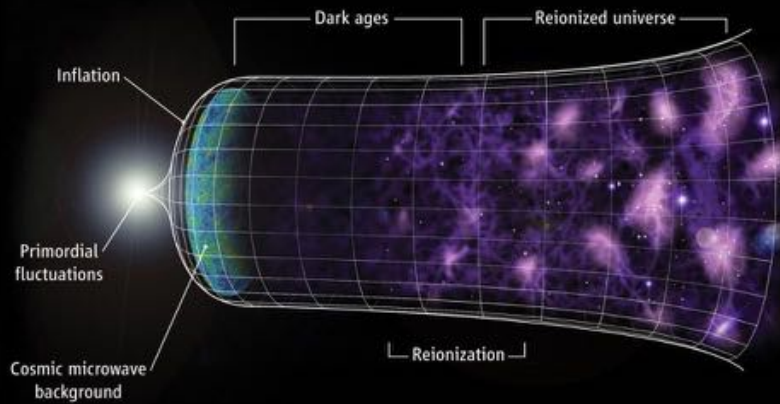
Cosmological Collider Physics

Zhong-Zhi Xianyu [鲜于中之]

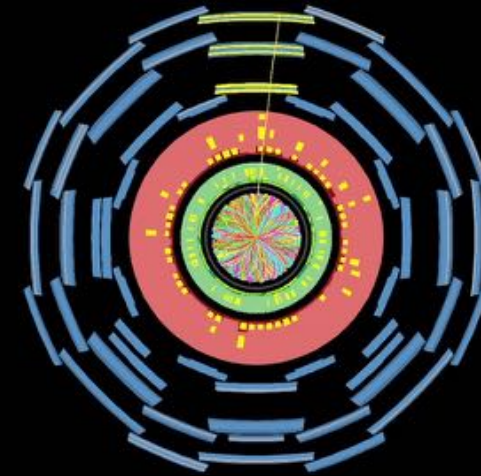
Department of Physics, Tsinghua University

Pre-SUSY Summer School

August 10, 2021 · Beijing



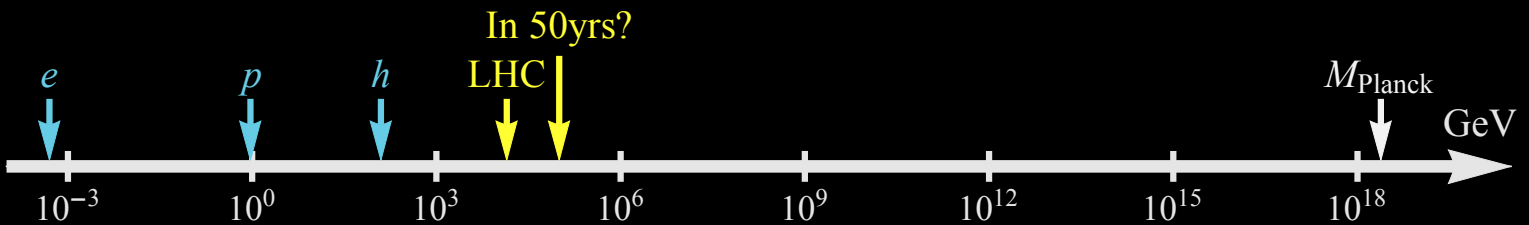
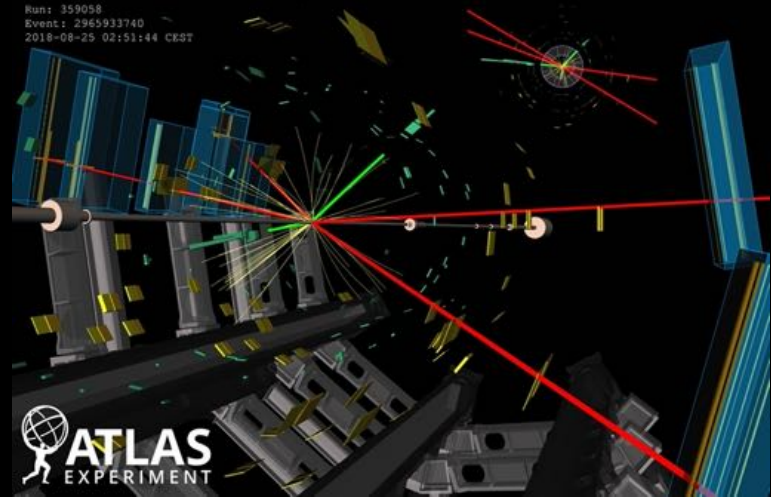
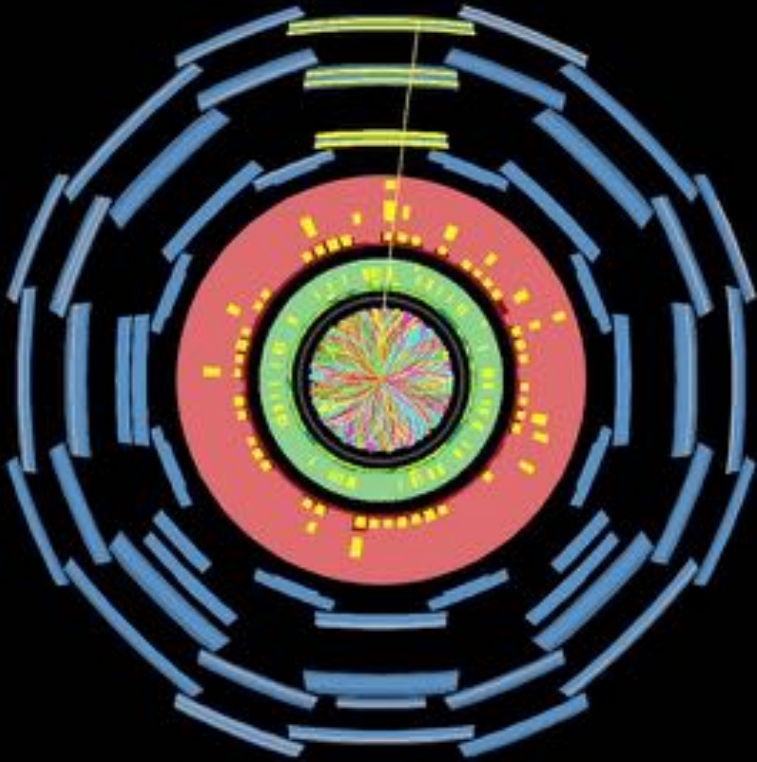
We've learnt cosmology



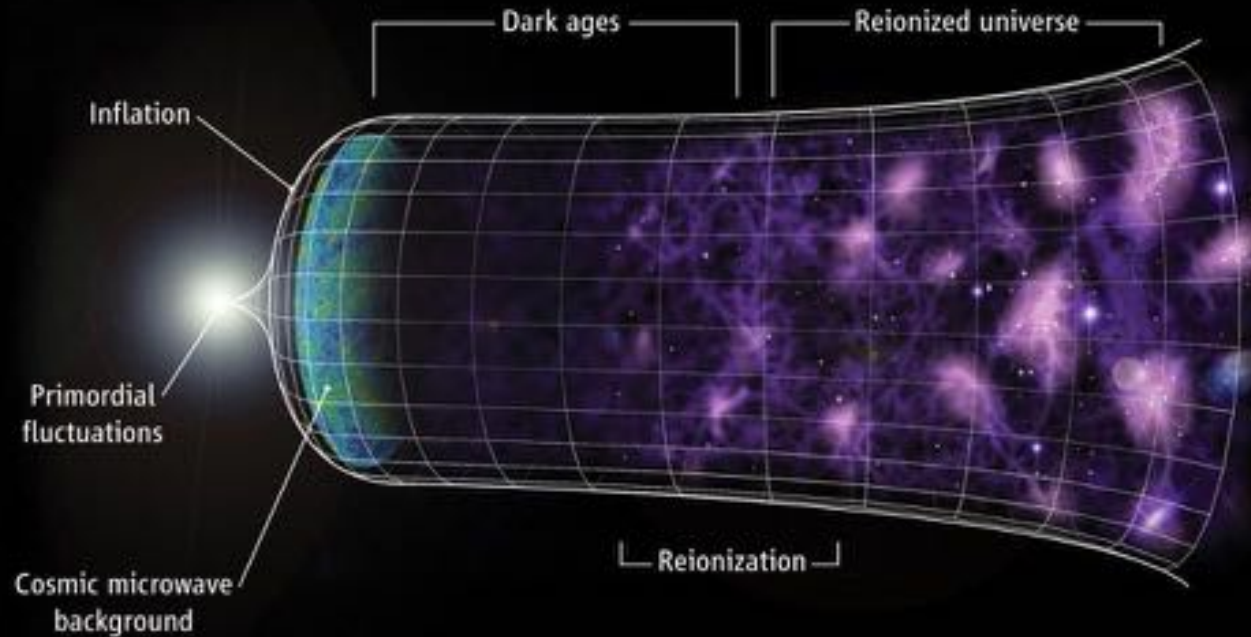
We've learnt collider physics

Now, what is a **cosmological collider**?

Collider physics



Cosmic inflation

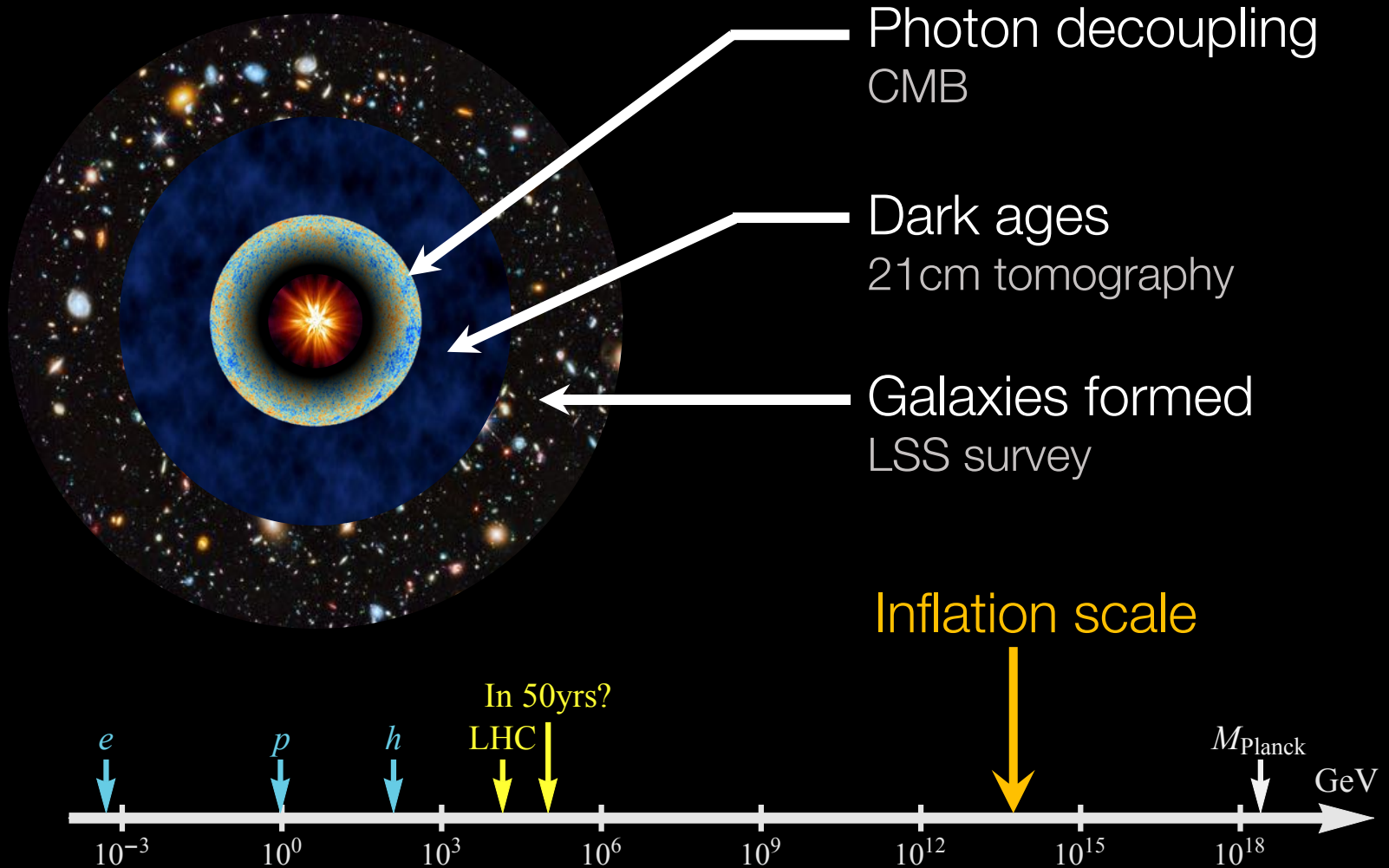


The scale factor $a(t)$	e^{Ht}	inflation	$H \sim 10^{14} \text{ GeV}$
	$t^{1/2}$	radiation domination	
	$t^{2/3}$	matter domination	

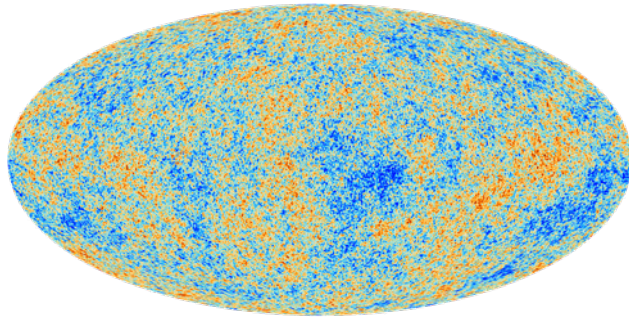


前方高能

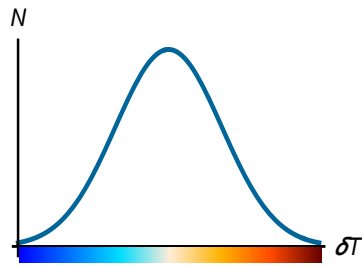
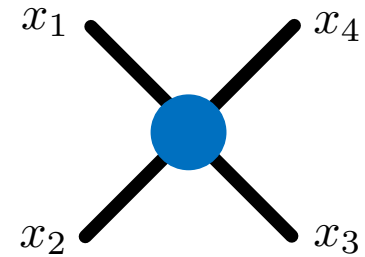
Cosmological Collider



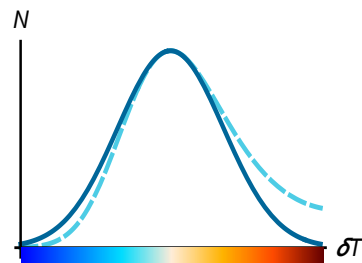
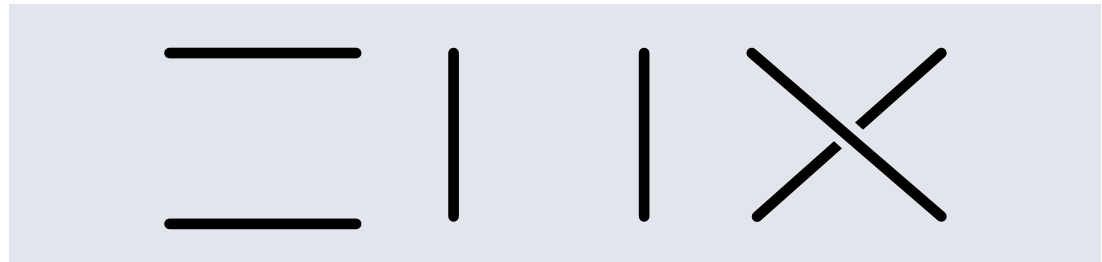
How to extract information from CMB map?



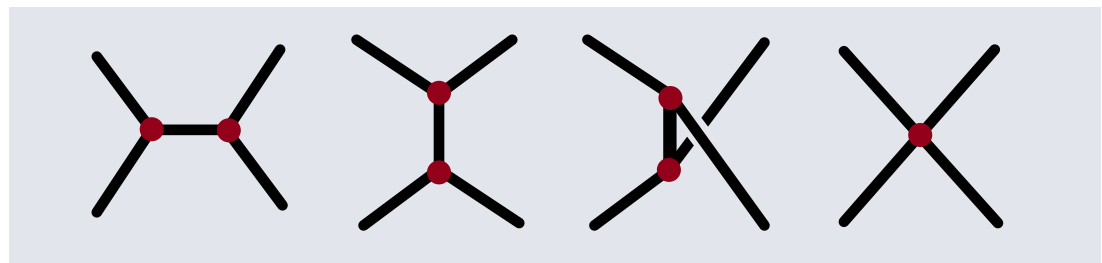
$$\langle \delta T(x_1) \cdots \delta T(x_n) \rangle$$



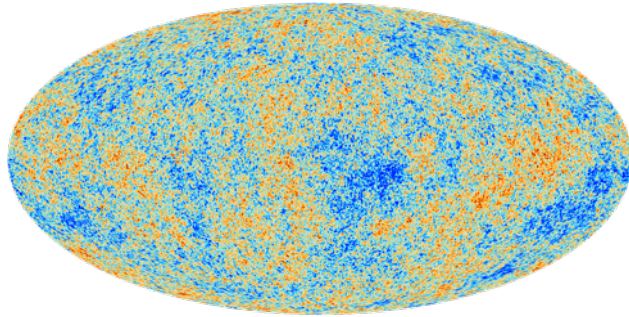
Gaussian



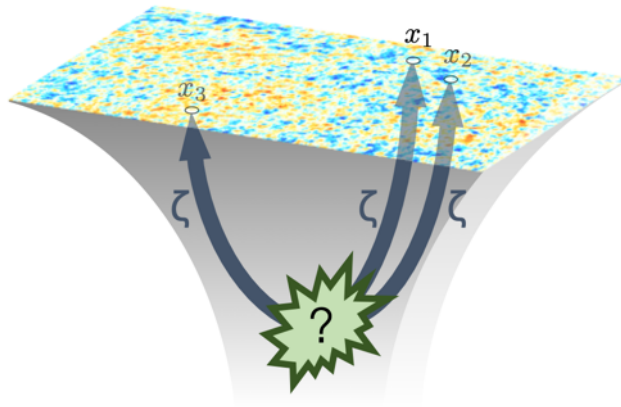
Non-Gaussian



What's the physics behind?



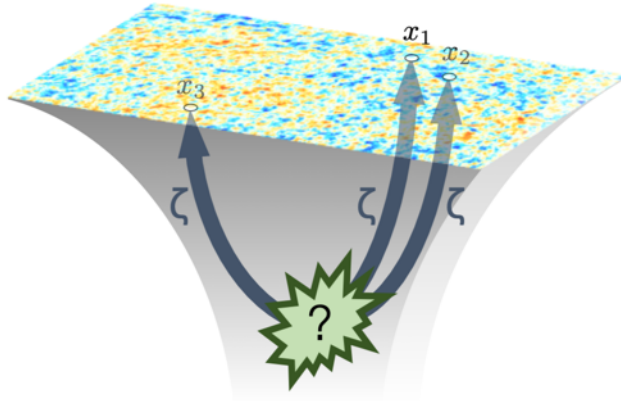
During the inflation, the expansion rate is fast enough to produce particles in pairs purely from quantum fluctuations.



The fluctuations were then redshifted and “frozen” at large scales and seeded the density perturbation that we see today.

Non-Gaussianity ~ interaction

Discover new heavy particles



When massive particles are produced, the inflation did two things:

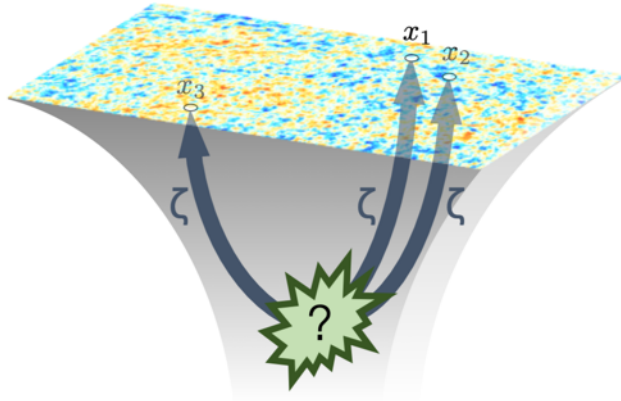
1. Dilute the number density
2. Exhaust the momentum, so that the particle quickly becomes nonrelativistic

$$\sigma(t) \sim (Ae^{+imt} + Be^{-imt})e^{-\frac{3}{2}Ht}$$

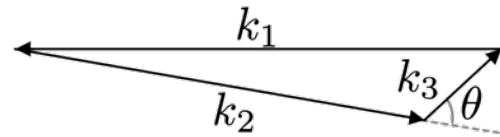
We would be able to measure the mass
if we can trace the time dependence

But we can't. We observe only the final state
[e.g. through CMB]

Discover new heavy particles



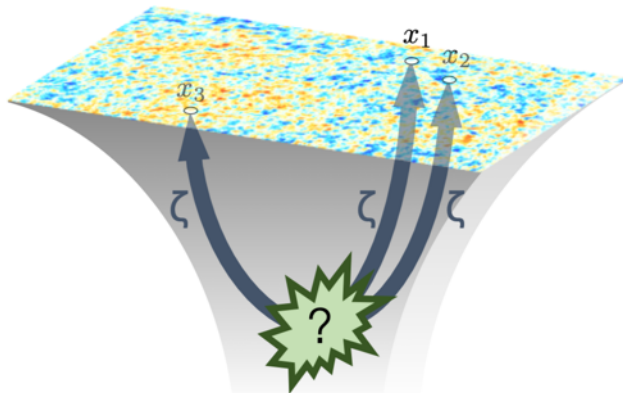
A solution: we try to measure the 3-point correlation in the **squeezed limit**



Small-momentum mode redshifts **earlier**, and oscillates like a **nonrelativistic particle** when the other two large-momentum modes are still deeply inside the horizon

The ratio of long and short momenta is actually a measure of time difference => Measure the 3pt function at different k ratio ~ measure the mode at different time

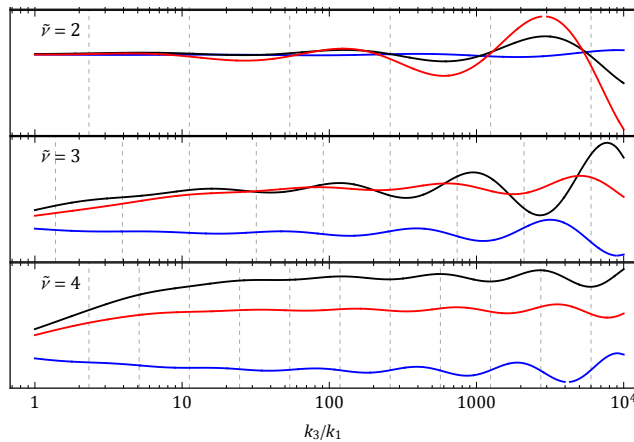
Discover new heavy particles



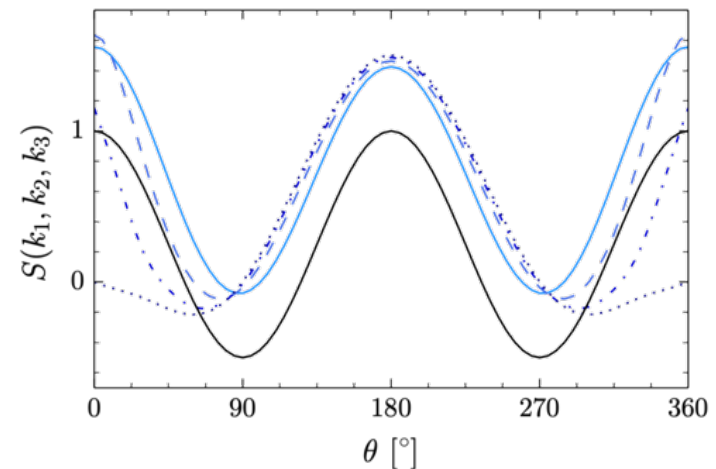
Chen, Wang, 0911.3380;1205.0160
Arkani-Hamed, Maldacena, 1503.08043

$$S(\mathbf{k}_1, \mathbf{k}_3) = A(\lambda, m) \left(\frac{k_3}{k_1} \right)^{1/2 \pm \nu} P_s(\cos \theta)$$

$$\nu = \begin{cases} \sqrt{\frac{9}{4} - \frac{m^2}{H^2}} & s = 0 \\ \sqrt{\left(s - \frac{1}{2}\right)^2 - \frac{m^2}{H^2}} & s \neq 0 \end{cases}$$



Chen, Chua, Guo, Wang, ZZX, Xie, 1803.04412

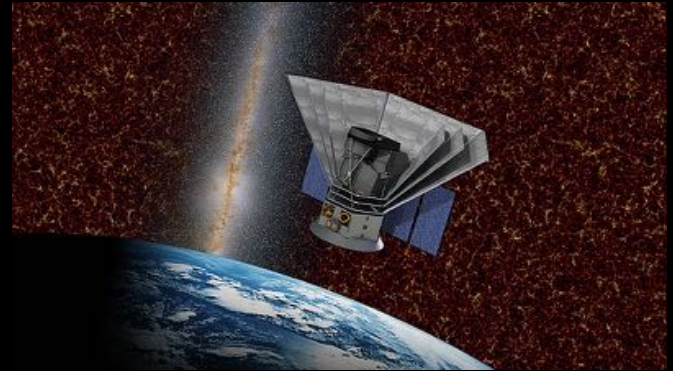


Lee, Baumann, Pimentel, 1607.03735

Why now?



Planck: final data release in 2018



SPHEREx: selected by NASA in 2019, launching in ~2024

Planck 2018

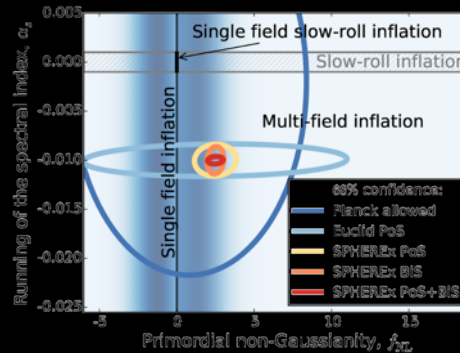
1905.05697

$$f_{\text{NL}}^{(\text{local})} = -0.9 \pm 5.1$$

$$f_{\text{NL}}^{(\text{equil})} = -26 \pm 47$$

$$f_{\text{NL}}^{(\text{ortho})} = -38 \pm 24$$

O(1) in ~10yrs?

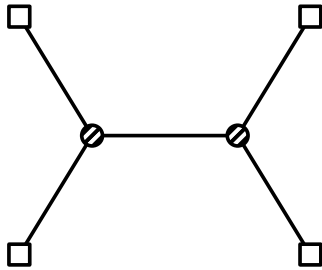


SPHEREx, 1412.4872

O(0.01) ultimately
21cm tomography

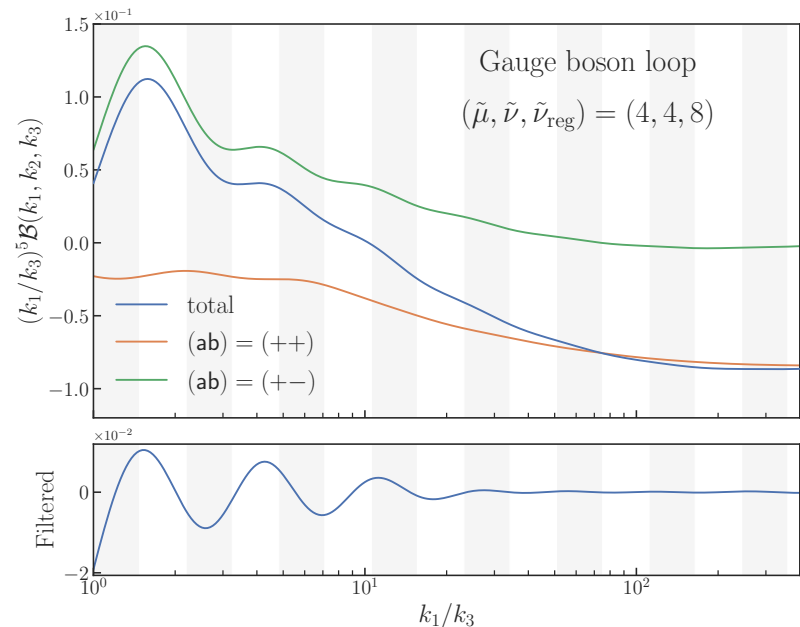
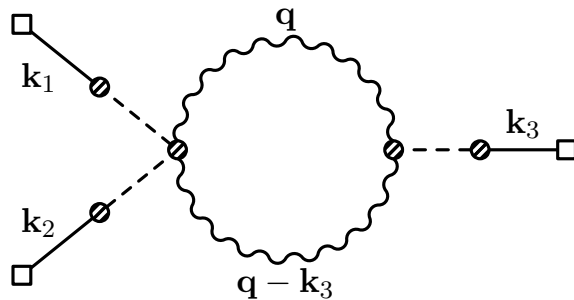
Meerburg, Muñoz, Ali-Haïmoud, Kamionkowski, 1506.04152; Münchmeyer, Muñoz, Chen, 1610.06559; Dizgah, Lee, Muñoz, Dvorkin 1801.07265;

Why now?



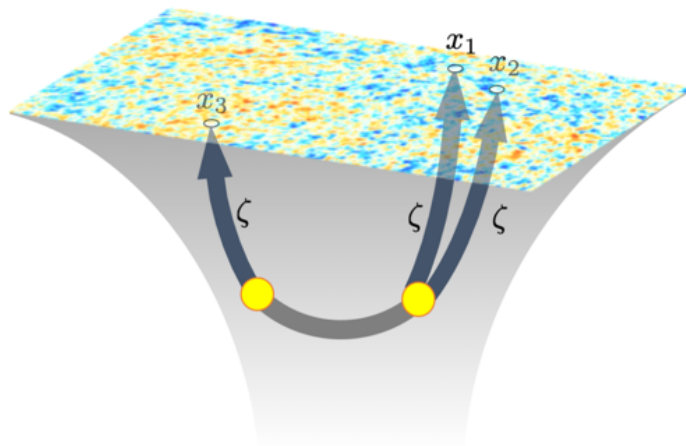
Computing cosmic correlators are challenging!
Feynman diagram in de Sitter space
Technical difficulties + conceptual problems

Many new developments recently
in both formal and phenomenological aspects



Lian-Tao Wang, ZZX, Yi-Ming Zhong, to appear

Outline



Background evolution

Perturbation theory

Quantum fields in dS

Schwinger-Keldysh formalism

Cosmic correlators

Phenomenology

Useful references

Cosmology in general

- Dodelson: *Modern cosmology*
- Weinberg: *Cosmology*
- Mukhanov: *Physical Foundations of Cosmology*

Inflation and non-Gaussianity

- Baumann & McAllister 1404.2601
- Baumann: 0907.5424
- Maldacena: astro-ph/0210603
- Chen: 1002.1416
- Wang: 1303.1523

QFT in de Sitter space

- Spradlin, Strominger Volovich: hep-th/0110007
- Akhmedov: 1309.2557

Schwinger-Keldysh formalism

- Jordan: PRD 33 (1986) 444
- Chou, Su, Hao, Yu: Phys. Rept. 118 (1985) 1
- Chen, Wang, ZZX: 1703.10166

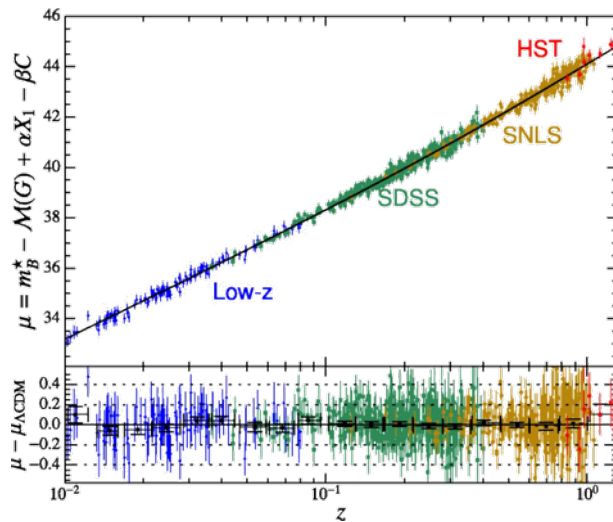
Cosmological collider physics

- Arkani-Hamed & Maldacena: 1503.08043
- Lee, Baumann, Pimentel: 1607.03735
- Wang, ZZX: 1910.12876

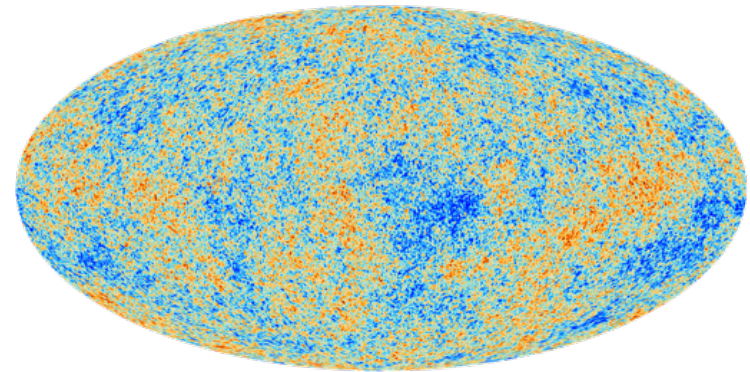
The need for cosmic inflation

Two basic facts

1. The universe is expanding
2. The universe is homogenous & isotropic at large scales



Betoule M, et al. Astron Astrophys 568 (2014) 22



Planck

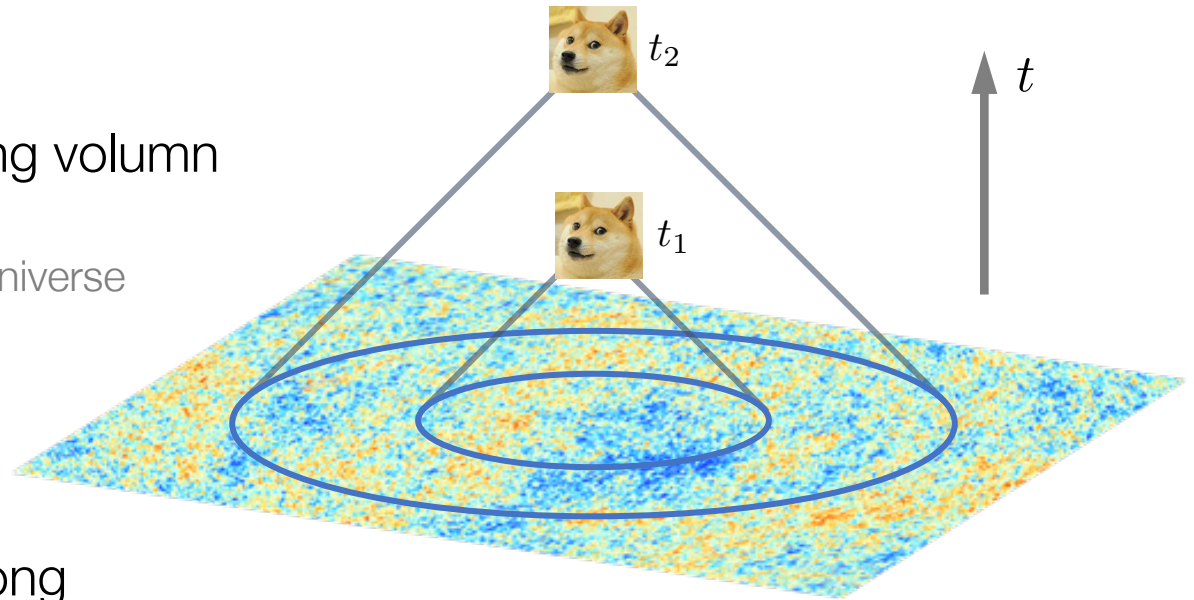
The need for cosmic inflation

Horizon problem

We see larger comoving volume later than earlier

[Still true for an expanding universe filled with ordinary matter]

Why CMB sky is so uniform if there was no causal contact among different patches?



The need for cosmic inflation

Horizon problem

We see larger comoving volume later than earlier

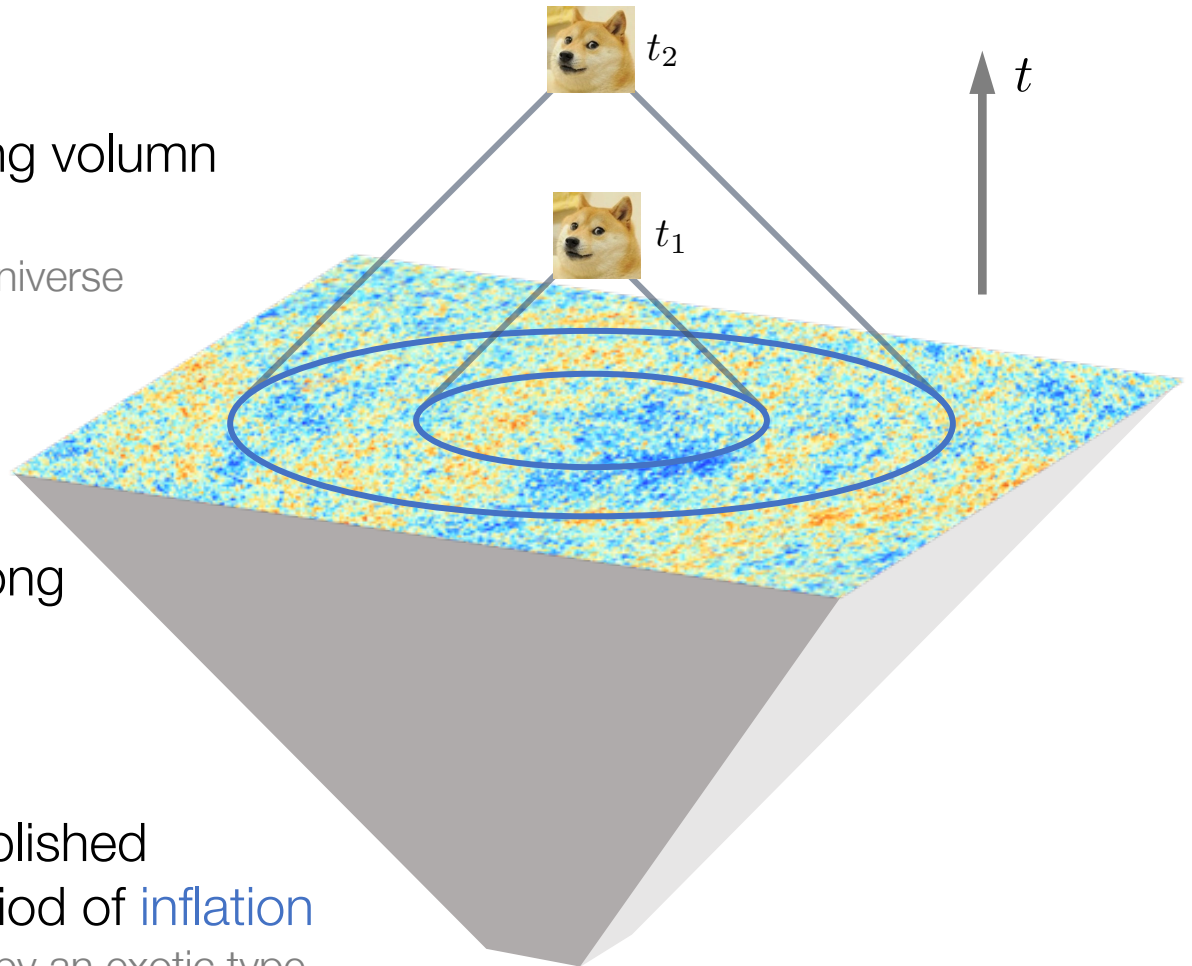
[Still true for an expanding universe filled with ordinary matter]

Why CMB sky is so uniform if there was no causal contact among different patches?

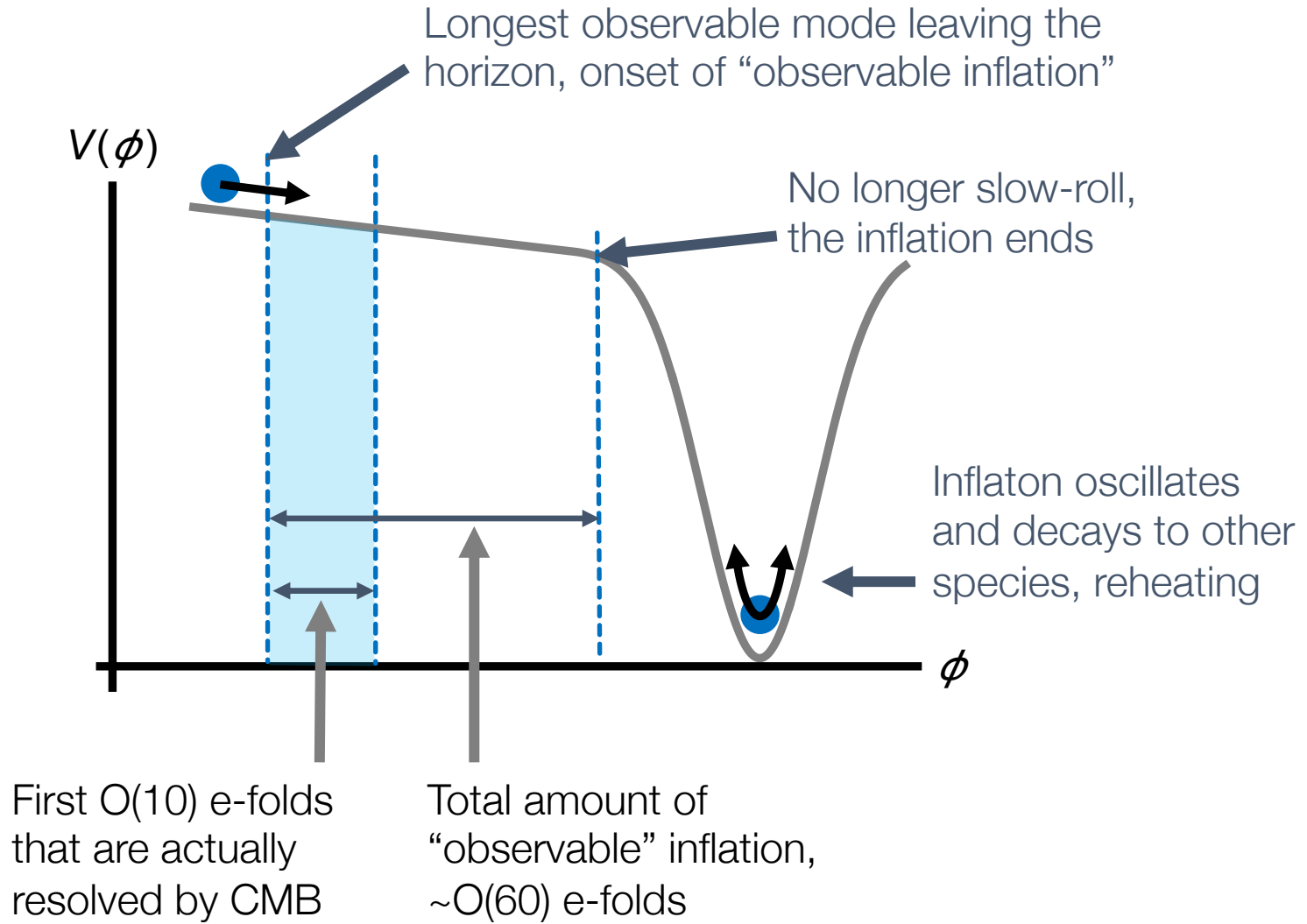
Solution

Causal contact reestablished if there has been a period of **inflation**

[fast expansion, dominated by an exotic type of energy, rather than ordinary matter]



Background dynamics



Background dynamics

Einstein gravity + a scalar supporting inflation (inflaton)

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R - \frac{1}{2} (\partial_\mu \phi)^2 - V(\phi) \right]$$

Homogeneity and isotropy dictate, at background level,

$$ds^2 = -dt^2 + a^2(t) d\mathbf{x}^2 \quad \phi = \phi_0(t)$$

$$\ddot{\phi} + 3H\dot{\phi} + V'_\phi(\phi) = 0$$

$$3M_{\text{Pl}}^2 H^2(t) = \rho(t) \quad H \equiv \dot{a}/a$$

$$\dot{\rho} + 3H(\rho + p) = 0$$

$$M_{\text{Pl}} = (8\pi G)^{-1/2} \simeq 2.4 \times 10^{18} \text{TeV}$$

$$\rho = \frac{1}{2} \dot{\phi}^2 + V(\phi)$$

$$p = \frac{1}{2} \dot{\phi}^2 - V(\phi)$$

The inflaton should roll slowly, described by slow-roll parameters

$$\epsilon = \frac{\dot{\phi}^2}{2M_{\text{Pl}}^2 H^2}$$

$$\eta = \frac{\ddot{\phi}}{H\dot{\phi}}$$

$$\eta \sim \mathcal{O}(0.01)$$

$$\epsilon < \mathcal{O}(0.003)$$

Perturbation theory

Einstein gravity + a scalar supporting inflation (inflaton)

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R - \frac{1}{2} (\partial_\mu \phi)^2 - V(\phi) \right]$$

$$\phi(t, \mathbf{x}) = \phi_0(t) + \varphi(t, \mathbf{x})$$

10 dofs in the metric,
one in the inflaton

$$ds^2 = -N^2 dt^2 + h_{ij} (dx^i + N^i dt)(dx^j + N^j dt)$$

- 4 gauge dofs, 4
constrained variables

$$N = 1 + \alpha$$

$$N_i = \partial_i \psi + \beta_i$$

= 3 physical dofs

$$h_{ij} = a^2 e^{2\zeta} (\delta_{ij} + \gamma_{ij} + \partial_i \kappa_j + \partial_j \kappa_i + \partial_i \partial_j \lambda)$$

1 scalar & 2 tensor

Scalar perturbation action to 2nd order [try yourself if you wish]

$$S_2 = M_{\text{Pl}}^2 \int dt d^3x \epsilon \left[a^3 \dot{\zeta}^2 - a (\partial_i \zeta)^2 \right]$$

Perturbation theory

Using conformal time τ

$$dt = a(\tau)d\tau$$

$$S_2 = \frac{1}{2} \int d\tau d^3x z^2 \left[(\zeta')^2 - (\partial_i \zeta)^2 \right] \quad \zeta' = d\zeta/d\tau \quad z \equiv \sqrt{2\epsilon} M_{\text{Pl}} a$$

Introducing a canonically normalized field $u = z\zeta$

$$S_2 = \frac{1}{2} \int d\tau d^3x \left[(u')^2 - (\partial_i u)^2 + \frac{z''}{z} u^2 \right]$$

Go to momentum space => “mode function”

$$u(\tau, \mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \left[u_k(\tau) e^{+i\mathbf{k}\cdot\mathbf{x}} + \text{c.c.} \right]$$

The general solution in a formal dS limit $\epsilon \rightarrow 0$ $a = e^{Ht} = -1/(H\tau)$

$$u_k = \frac{1}{\sqrt{2k}} \left[c_1 \left(1 - \frac{i}{k\tau} \right) e^{-ik\tau} + c_2 \left(1 + \frac{i}{k\tau} \right) e^{+ik\tau} \right]$$

Perturbation theory

Quantization

$$u(\tau, \mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \left[u_k(\tau) e^{+i\mathbf{k}\cdot\mathbf{x}} a_{\mathbf{k}} + u_k^*(\tau) e^{-i\mathbf{k}\cdot\mathbf{x}} a_{\mathbf{k}}^\dagger \right]$$

Two conditions to determine c_1 and c_2 :

1. canonical commutator (**Wronskian**)
2. Vacuum at early time limit (**Bunch-Davies vacuum**)

$$u_k = e^{i\theta} \frac{1}{\sqrt{2k}} \left(1 - \frac{i}{k\tau} \right) e^{-ik\tau}$$

In terms of the original fluctuation field

$$\zeta_k(\tau) = e^{i\theta} \frac{H}{2M_{\text{Pl}} \sqrt{\epsilon k^3}} (1 + ik\tau) e^{-ik\tau}$$

Perturbation theory

$$\zeta_k(\tau) = e^{i\theta} \frac{H}{2M_{\text{Pl}}\sqrt{\epsilon}k^3} (1 + ik\tau)e^{-ik\tau}$$

$$\tau \in (-\infty, 0)$$

Remarks:

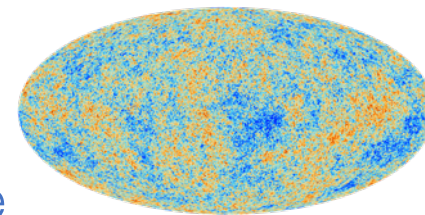
Early-time limit: plane waves of a massless field

[deep inside the horizon $\sim$ flat space]

Late-time limit: reaches a constant

[conserved at superhorizon, irrespective to small scale physics]

A complex phase unfixed, **stochastic quantum noise**

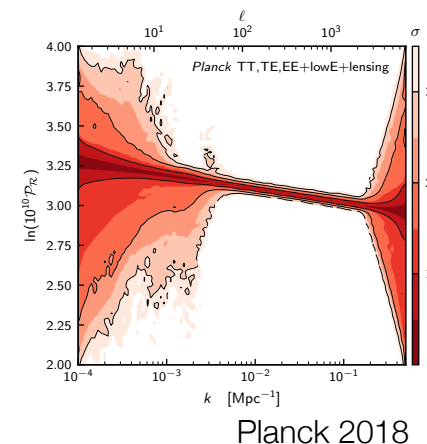


The complex phase cancels out in 2-point correlator [**power spectrum**]

$$\lim_{\tau \rightarrow 0} \langle 0 | \zeta_{\mathbf{k}}(\tau) \zeta_{\mathbf{q}}(\tau) | 0 \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k} + \mathbf{q}) \frac{2\pi^2}{k^3} \mathcal{P}_\zeta(k)$$

$$\mathcal{P}_\zeta(k) = \frac{H^2}{8\pi^2 \epsilon M_{\text{Pl}}^2} = \frac{H^4}{(2\pi)^2 \dot{\phi}_0^2} \quad \leftarrow \text{scale invariance}$$

$$\mathcal{P}_\zeta(k) \simeq 2 \times 10^{-9}$$



Quantum fields in dS

Very much similar, now consider an arbitrary **massive scalar field**

$$(\square - m^2)\sigma = 0 \quad \longrightarrow \quad \sigma''(\tau, \mathbf{x}) - \frac{2}{\tau}\sigma'(\tau, \mathbf{x}) - \partial_i^2\sigma(\tau, \mathbf{x}) + \frac{m^2}{H^2\tau^2}\sigma(\tau, \mathbf{x}) = 0$$

In momentum space, mode equation

$$\sigma_k''(\tau) - \frac{2}{\tau}\sigma_k'(\tau) + \left(k^2 + \frac{m^2}{H^2\tau^2}\right)\sigma_k(\tau) = 0$$

Unique normalized solution upon Bunch-Davies condition

$$\sigma_k(\tau) = \frac{\sqrt{\pi}}{2} e^{i\nu\pi/2} H(-\tau)^{3/2} \mathbf{H}_{\nu}^{(1)}(-k\tau) \quad \nu \equiv \sqrt{9/4 - (m/H)^2}$$



Hänkel function

You can already anticipate that $m < 3H/2$ and $m > 3H/2$ would behave very differently

Quantum fields in dS

$$\sigma_k(\tau) = \frac{\sqrt{\pi}}{2} e^{i\nu\pi/2} H(-\tau)^{3/2} H_\nu^{(1)}(-k\tau) \quad \nu \equiv \sqrt{9/4 - (m/H)^2}$$

Early-time limit: massless plain waves again [figure it out yourself]

Late-time limit:

$$\sigma_k(\tau) = -i\sqrt{\frac{2}{\pi k^3}} H \left[e^{-i\nu\pi/2} \Gamma(-\nu) \left(\frac{-k\tau}{2} \right)^{3/2+\nu} + (\nu \rightarrow -\nu) \right]$$

$m < 3H/2$ non-analytical scaling in $k\tau$ $m > 3H/2$ oscillatory in $k\tau$

In general: $\lim_{\tau \rightarrow 0} \sigma(\tau, \mathbf{x}) = \sigma_+(\mathbf{x})(-\tau)^{\Delta_+} + \sigma_-(\mathbf{x})(-\tau)^{\Delta_-}$, $\Delta_{\pm} = \frac{3}{2} \pm \nu$

The dS has isometry group $SO(4,1)$, under which the late time coefficient $\sigma_{\pm}$ transforms like a scalar of conformal weight Δ

The isometry in the bulk ~ conformal group on the future boundary

Only a symmetry statement! Any dynamical realization? [dS/CFT]

Quantum fields in dS

Consider $m \gg H$ limit:

$$\nu = i\tilde{\nu}$$

$$\sigma_{\mathbf{k}}(t) = \frac{1}{\sqrt{2\tilde{\nu}H}} e^{-3Ht/2} \left(e^{-i\tilde{\nu}Ht} b_{\mathbf{k}} + e^{+i\tilde{\nu}Ht} b_{-\mathbf{k}}^{\dagger} \right)$$

while at early times: $\sigma_{\mathbf{k}}(\tau) = \sigma_k a_{\mathbf{k}} + \sigma_k^* a_{-\mathbf{k}}^{\dagger}$

Two sets of operators linearly related $b_{\mathbf{k}} = \alpha_k a_{\mathbf{k}} + \beta_k a_{-\mathbf{k}}^{\dagger}$

[Bogoliubov transformation]

Computing the occupation number at late times:

$$n_{\mathbf{k}} = \langle 0 | b_{\mathbf{k}}^{\dagger} b_{\mathbf{k}} | 0 \rangle = |\beta_k|^2$$

$$|\beta_k|^2 = \frac{\tilde{\nu}}{2\pi} |\Gamma(-i\tilde{\nu})|^2 e^{-\tilde{\nu}\pi} = \frac{\coth \tilde{\nu}\pi - 1}{2} \simeq e^{-2\pi m/H}$$

Nonzero β_k signals particle creation

looks like thermal distribution $e^{-m/T}$

An engine for the cosmological collider

A first hint that dS vacuum is thermal, with temperature $T = \frac{H}{2\pi}$

Quantum fields in dS

Classification of states

UIRs of dS isometry group $SO(4,1)$

classified by **spin s** under $SO(3)$, and **conformal weight Δ** under $SO(1,1)$

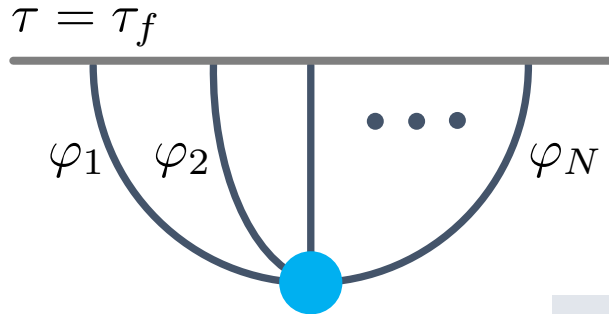
spin	$s = 0$	$s \in \frac{1}{2}\mathbb{Z}_+$
weight	$\Delta = \frac{3}{2} \pm \sqrt{\frac{9}{4} - (m/H)^2}$	$\Delta = \frac{3}{2} \pm \sqrt{(s - \frac{1}{2})^2 - (m/H)^2}$
principal series	$m > 3H/2$	$(m/H)^2 > (s - \frac{1}{2})^2$
complementary series	$0 < m < 3H/2$	$(m/H)^2 < (s - \frac{1}{2})^2$
discrete series		$(m/H)^2 = s(s-1) - t(t+1)$ $t = 0, 1, \dots, s-1$

Strickly speaking, massless scalar does not exist in dS

But both massless spin-1 and massless spin-2 are allowed

Schwinger-Keydish formalism

Goal: calculating equal-time correlators



$$\langle \Omega | \varphi^{A_1}(\tau, \mathbf{x}_1) \cdots \varphi^{A_N}(\tau, \mathbf{x}_N) | \Omega \rangle$$

$$1 = \sum |O_\alpha\rangle \langle O_\alpha|$$

Convert equal-time correlators into a pair of in-out amplitude, then apply standard path integral method

$$\begin{aligned} & \langle \Omega | \varphi^{A_1}(\tau, \mathbf{x}_1) \cdots \varphi^{A_N}(\tau, \mathbf{x}_N) | \Omega \rangle \\ &= \sum_{\alpha} \langle \Omega | O_{\alpha} \rangle \langle O_{\alpha} | \varphi^{A_1}(\tau, \mathbf{x}_1) \cdots \varphi^{A_N}(\tau, \mathbf{x}_N) | \Omega \rangle \\ &= \int \mathcal{D}\varphi_+ \mathcal{D}\varphi_- \varphi_+^{A_1}(\tau, \mathbf{x}_1) \cdots \varphi_+^{A_N}(\tau, \mathbf{x}_N) \\ & \quad \times \exp \left[i \int_{\tau_0}^{\tau_f} d\tau d^3\mathbf{x} \left(\mathcal{L}_{\text{cl}}[\varphi_+] - \mathcal{L}_{\text{cl}}[\varphi_-] \right) \right] \\ & \quad \times \prod_{A, \mathbf{x}} \delta \left(\varphi_+^A(\tau_f, \mathbf{x}) - \varphi_-^A(\tau_f, \mathbf{x}) \right) \end{aligned}$$

Schwinger-Keydish formalism

2 fields => 4 types of **bulk** propagators

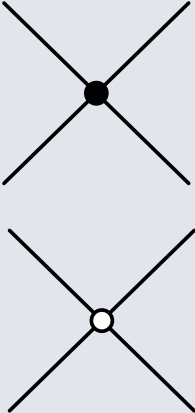
$$\begin{aligned}
 \overset{\tau_1}{\bullet} \text{---} \overset{\tau_2}{\bullet} &= G_{++}(k; \tau_1, \tau_2) \\
 &= u(\tau_1, k)u^*(\tau_2, k)\theta(\tau_1 - \tau_2) + u^*(\tau_1, k)u(\tau_2, k)\theta(\tau_2 - \tau_1) \\
 \overset{\tau_1}{\circ} \text{---} \overset{\tau_2}{\circ} &= G_{--}(k; \tau_1, \tau_2) \\
 &= u^*(\tau_1, k)u(\tau_2, k)\theta(\tau_1 - \tau_2) + u(\tau_1, k)u^*(\tau_2, k)\theta(\tau_2 - \tau_1) \\
 \overset{\tau_1}{\bullet} \text{---} \overset{\tau_2}{\circ} &= G_{+-}(k; \tau_1, \tau_2) = u^*(\tau_1, k)u(\tau_2, k) \\
 \overset{\tau_1}{\circ} \text{---} \overset{\tau_2}{\bullet} &= G_{-+}(k; \tau_1, \tau_2) = u(\tau_1, k)u^*(\tau_2, k)
 \end{aligned}$$

2 **bulk-to-boundary** propagators

$$\begin{aligned}
 \overset{\tau}{\bullet} \text{---} \square &= G_+(k; \tau) \equiv G_{++}(k; \tau, \tau_f) \\
 \overset{\tau}{\circ} \text{---} \square &= G_-(k; \tau) \equiv G_{-+}(k; \tau, \tau_f)
 \end{aligned}$$

Schwinger-Keydish formalism

2 types of vertices

$$\mathcal{L}_{\text{int}} \supset -\frac{\lambda}{24} a^4(\tau) \varphi^4$$

$$= -i\lambda \int_{\tau_0}^{\tau_f} d\tau a^4(\tau) \dots$$
$$= +i\lambda \int_{\tau_0}^{\tau_f} d\tau a^4(\tau) \dots$$

Feynman rule

Draw **square** to each final endpoint; draw **circle** to each vertex

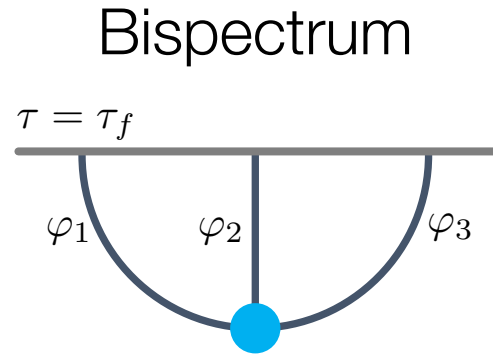
Color vertices in either **white** or **black** in **all possible ways**

Integrate over all **unconstrained (loop) momenta**

Integrate over the **time variable** at each **vertex**

Sum over all possible coloring

Cosmic correlators



$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle' = \frac{(2\pi)^4 \mathcal{P}_\zeta^2}{(k_1 k_2 k_3)^2} \mathcal{S}(k_1, k_2, k_3)$$

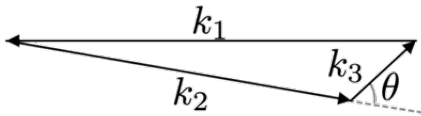
Momentum conservation δ stripped

Power spectrum
 $P_\zeta \simeq 2 \times 10^{-9}$

Shape function
 dimensionless and
 (usually) scale invariant

Conversion between ζ and φ

$$\zeta = -\frac{H}{\dot{\phi}_0} \varphi$$



The 3-point function depends only on the **shape** of the momentum triangle, [3d translation]
 not on the orientation, [3d rotation]
 not on the size [scale symmetry]

Cosmic correlators

3-point correlator from the inflaton:
expanding the action to 3rd order
[don't try yourself unless you really wish!]

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R - \frac{1}{2} (\partial_\mu \phi)^2 - V(\phi) \right]$$

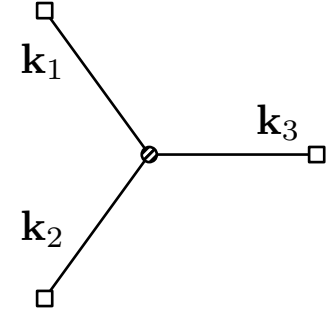
$$S_2 = M_{\text{Pl}}^2 \int dt d^3x \epsilon \left[a^3 \dot{\zeta}^2 - a (\partial_i \zeta)^2 \right]$$

$$S_3 = M_{\text{Pl}}^2 \int d\tau d^3\mathbf{x} \left\{ a^2 \epsilon^2 \left[\zeta (\zeta')^2 + \zeta (\partial_i \zeta)^2 - 2 \zeta' \partial_i \zeta \partial_i^{-1} \zeta' \right] \right. \\ \left. - \frac{1}{2} \partial_\tau \left(\epsilon \eta a^2 \zeta^2 \zeta' \right) \right\} + \dots$$

Dots represent higher orders in slow-roll parameters

Cosmic correlators

$$S_3 = M_{\text{Pl}}^2 \int d\tau d^3\mathbf{x} \left\{ a^2 \epsilon^2 \left[\zeta(\zeta')^2 + \zeta(\partial_i \zeta)^2 - 2\zeta' \partial_i \zeta \partial_i^{-1} \zeta' \right] - \frac{1}{2} \partial_\tau \left(\epsilon \eta a^2 \zeta^2 \zeta' \right) \right\} + \dots$$



$$\begin{aligned} & \langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle'_{(1)} \\ &= 2i M_{\text{Pl}}^2 \epsilon^2 \sum_{\mathbf{a}=\pm} \mathbf{a} \int_{-\infty}^{\tau_f} \frac{d\tau}{(H\tau)^2} G_{\mathbf{a}}(k_1, \tau) \partial_\tau G_{\mathbf{a}}(k_2, \tau) \partial_\tau G_{\mathbf{a}}(k_3, \tau) + 2 \text{ perms} \\ &= -4 M_{\text{Pl}}^2 \epsilon^2 \text{Im} \left[\prod_{i=1}^3 \zeta_{k_i}(\tau_f) \int_{-\infty}^{\tau_f} \frac{d\tau}{(H\tau)^2} \zeta_{k_1}^*(\tau) \zeta_{k_2}'^*(\tau) \zeta_{k_3}'^*(\tau) \right] + 2 \text{ perms} \\ &= -\frac{H^4}{16 M_{\text{Pl}}^4 \epsilon k_1^3 k_2 k_3} \text{Im} \int_{-\infty}^{\tau_f} d\tau (1 - i k_1 \tau) e^{+i k_t \tau} + 2 \text{ perms} \\ &= \frac{H^4}{4 M_{\text{Pl}}^4 \epsilon k_1^3 k_2 k_3 k_t} + 2 \text{ perms} \end{aligned}$$

$$k_t \equiv k_1 + k_2 + k_3$$

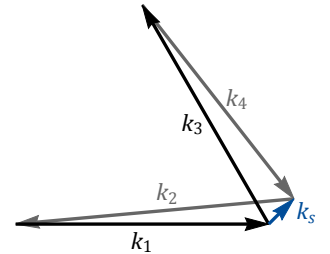
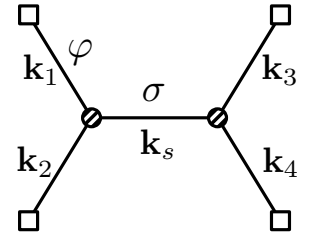
$$\mathcal{S} = \epsilon \left(\frac{k_1 k_2}{k_3 k_t} + 2 \text{ perms} \right) + \frac{\epsilon}{8} \left(\frac{k_1}{k_2} + 5 \text{ perms} \right) + \frac{\eta - \epsilon}{8} \left(\frac{k_1^2}{k_2 k_3} + 2 \text{ perms} \right)$$

The bispectrum from the inflaton+gravity is **slow-roll suppressed**

Cosmic correlators

Consider now a cosmological collider process:
a 4-point function mediated by a massive scalar σ

$$a^2(\varphi')^2\sigma/\Lambda \subset \sqrt{-g}(\partial_\mu\phi)^2\sigma/\Lambda \subset \mathcal{L}$$



$$\begin{aligned} & \langle \varphi_{\mathbf{k}_1} \varphi_{\mathbf{k}_2} \varphi_{\mathbf{k}_3} \varphi_{\mathbf{k}_4} \rangle' \\ &= \frac{1}{\Lambda^2} \sum_{a,b=\pm} ab \int_{-\infty}^0 \frac{d\tau_1}{(-H\tau_1)^2} \frac{d\tau_2}{(-H\tau_2)^2} \quad \begin{array}{c} \varphi \\ \downarrow \end{array} \quad \begin{array}{c} \sigma \\ \downarrow \end{array} \\ & \quad \times G'_a(k_1, \tau_1) G'_a(k_2, \tau_1) G'_b(k_3, \tau_2) \underbrace{G'_b(k_4, \tau_2)}_{\text{blue}} \underbrace{D_{ab}(k_s, \tau_1, \tau_2)}_{\text{pink}} \\ &= \frac{1}{\Lambda^2} \frac{1}{16k_1 k_2 k_3 k_4} \sum_{a,b=\pm} ab \int_{-\infty}^0 d\tau_1 d\tau_2 e^{ia k_{12} \tau_1 + ib k_{34} \tau_2} D_{ab}(k_s; \tau_1, \tau_2) \end{aligned}$$

In the collapsed limit, the late-time expansion is valid

The intermediate soft mode well outside the horizon

$$D_{>}(k; \tau_1, \tau_2) = \sigma_k(\tau_1) \sigma_k^*(\tau_2) \sim D_1(k; \tau_1, \tau_2) + D_{\text{nl}}(k; \tau_1, \tau_2)$$

$$D_1(k; \tau_1, \tau_2) = \frac{H^2}{4\pi} (\tau_1 \tau_2)^{3/2} \Gamma(-i\tilde{\nu}) \Gamma(i\tilde{\nu}) \left[e^{\tilde{\nu}\pi} (\tau_1/\tau_2)^{i\tilde{\nu}} + e^{-\tilde{\nu}\pi} (\tau_1/\tau_2)^{-i\tilde{\nu}} \right]$$

$$D_{\text{nl}}(k; \tau_1, \tau_2) = \frac{H^2}{4\pi} (\tau_1 \tau_2)^{3/2} \left[\Gamma^2(-i\tilde{\nu}) (k^2 \tau_1 \tau_2)^{+i\tilde{\nu}} + \Gamma^2(+i\tilde{\nu}) (k^2 \tau_1 \tau_2)^{-i\tilde{\nu}} \right]$$

Cosmic correlators

$$\begin{aligned} \int_{-\infty}^0 d\tau_1 (-\tau_1)^{3/2} e^{i\mathbf{a}k_{12}\tau_1} (-k_s \tau_1)^{i\tilde{\nu}} &= \frac{1}{k_{12}^{5/2}} \left(\frac{k_s}{k_{12}} \right)^{i\tilde{\nu}} \int_0^{\infty} dz z^{3/2+i\tilde{\nu}} e^{-iaz} \\ &= e^{-i\mathbf{a}\pi/4 + \mathbf{a}\mathbf{c}\pi\tilde{\nu}/2} \Gamma\left(\frac{5}{2} + i\tilde{\nu}\right) \frac{1}{k_{12}^{5/2}} \left(\frac{k_s}{k_{12}} \right)^{i\tilde{\nu}} \end{aligned}$$

$$\int_{-\infty}^0 d\tau_2 (-\tau_2)^{3/2} e^{i\mathbf{b}k_{34}\tau_2} (-k_s \tau_2)^{i\tilde{\nu}} = e^{-i\mathbf{b}\pi/4 + \mathbf{b}\mathbf{c}\pi\tilde{\nu}/2} \Gamma\left(\frac{5}{2} + i\tilde{\nu}\right) \frac{1}{k_{34}^{5/2}} \left(\frac{k_s}{k_{34}} \right)^{i\tilde{\nu}}$$

$$\begin{aligned} \langle \varphi_{\mathbf{k}_1} \varphi_{\mathbf{k}_2} \varphi_{\mathbf{k}_3} \varphi_{\mathbf{k}_4} \rangle'_{\text{nl}} &\simeq \\ -\frac{H^2}{32\pi\Lambda^2} \frac{1}{k_1 k_2 k_3 k_4} \frac{1}{(k_{12} k_{34})^{5/2}} \text{Re} \left[\Gamma^2(-i\tilde{\nu}) \Gamma^2\left(\frac{5}{2} + i\tilde{\nu}\right) (1 + 2i \sinh \pi\tilde{\nu}) \left(\frac{k_s^2}{k_{12} k_{34}} \right)^{i\tilde{\nu}} \right] \end{aligned}$$

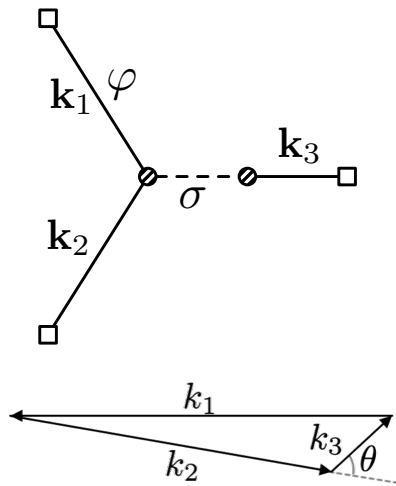
$$\langle \varphi_{\mathbf{k}_1} \varphi_{\mathbf{k}_2} \varphi_{\mathbf{k}_3} \varphi_{\mathbf{k}_4} \rangle'_{\text{nl}} \simeq \mathcal{A}(m, \Lambda) \frac{1}{k_1 k_2 k_3 k_4 (k_{12} k_{34})^{5/2}} \sin \left[\tilde{\nu} \log \left(\frac{k_s^2}{k_{12} k_{34}} \right) + \theta(\tilde{\nu}) \right]$$

$$\mathcal{A} = \frac{H^2}{32\pi\Lambda^2} \left| \Gamma^2(-i\tilde{\nu}) \Gamma^2\left(\frac{5}{2} + i\tilde{\nu}\right) (1 + 2i \sinh \pi\tilde{\nu}) \right|$$

Oscillatory dependence
on momentum ratio

$$\mathcal{A} \simeq \frac{\pi H^2}{8\Lambda^2} \left(\frac{m}{H} \right)^3 e^{-\pi m/H} \leftarrow \text{Boltzmann suppressed for large mass}$$

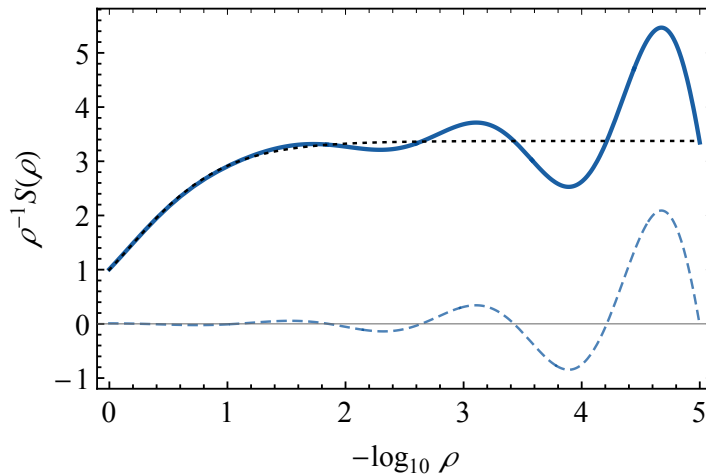
Cosmic correlators



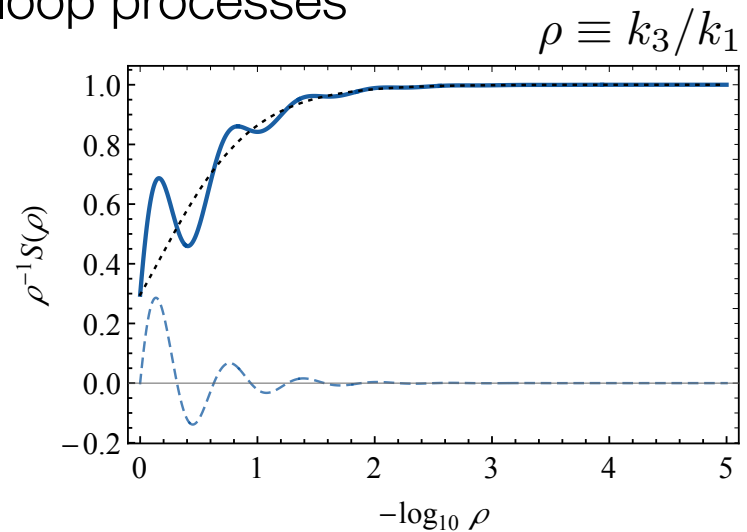
Bispectrum: similar but more difficult to calculate
In the squeezed limit, the signal part is

$$\lim_{k_3/k_1 \rightarrow 0} \mathcal{S}(k_1, k_2, k_3) \propto \frac{\dot{\phi}_0}{\Lambda^2} \left(\frac{m}{H} \right)^{3/2} e^{-\pi m/H} \left(\frac{k_1}{k_3} \right)^{-1/2} \sin \left(\tilde{\nu} \log \frac{k_1}{k_3} + \varphi \right)$$

Can generalize to loop processes

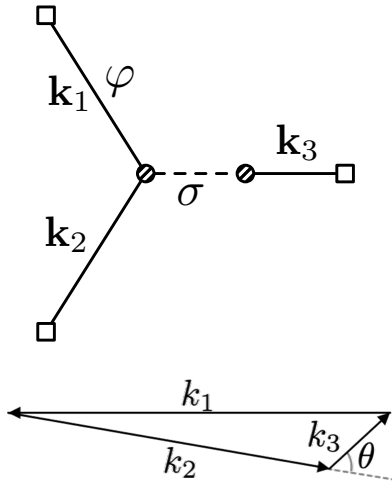


Tree process



Loop process

Cosmic correlators



Angular dependence in the squeezed limit

For intermediate states with mass m and spin s described by a symmetry tensor of rank s

$$\mathcal{O}_2 \sim \psi_{i_1 \dots i_s} \partial^{i_1} \dots \partial^{i_s} \varphi$$

$$\mathcal{O}_3 \sim \psi_{i_1 \dots i_s} (\partial^{i_1} \dots \partial^{i_n} \varphi) (\partial^{i_{n+1}} \dots \partial^{i_s} \varphi)$$

The 3-point vertex generates, in the squeezed limit,

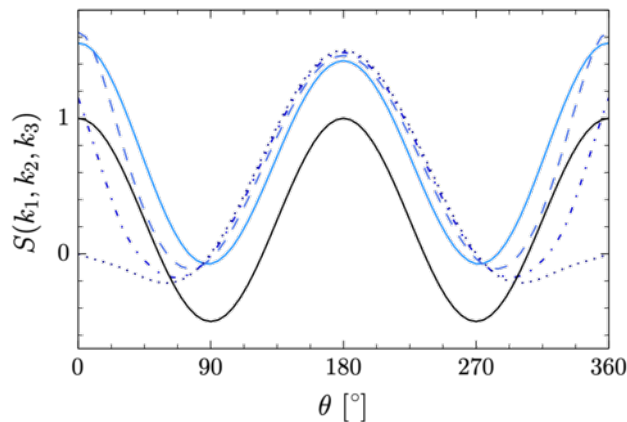
$$\epsilon_{i_1 \dots i_s}^0(\mathbf{k}_3) k_1^{i_1} \dots k_1^{i_s} \longrightarrow \epsilon_{i_1 \dots i_s}^0(\mathbf{k}_3) \epsilon_{i_1 \dots i_s}^{0*}(\mathbf{k}_1)$$

$$\mathbf{k}_1 \simeq -\mathbf{k}_2$$

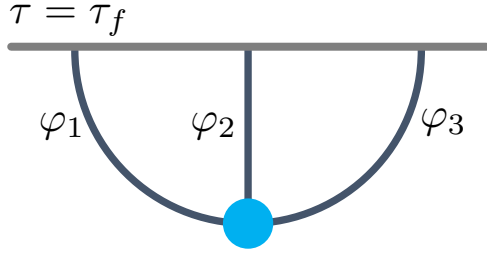
Longitudinal polarization tensor
symmetric and traceless

Matrix element of 3d rotation bringing $\mathbf{k}_1$ to $\mathbf{k}_3$
between two states of spin- s and helicity-0

$$\epsilon_{i_1 \dots i_s}^0(\mathbf{k}_3) k_1^{i_1} \dots k_1^{i_s} \propto P_s(\hat{\mathbf{k}}_1 \cdot \hat{\mathbf{k}}_3)$$



Phenomenology



Cosmological collider observables @ 3pt level
Angular dependence not shown

$$\rho \equiv k_3/k_1$$

$$\lim_{\varrho \rightarrow 0} \mathcal{S}(\varrho) = \underbrace{A\varrho^N [1 + \mathcal{O}(\varrho)]}_{\text{analytic}} + \underbrace{B\varrho^L [\sin(\alpha \log \varrho + \varphi) + \mathcal{O}(\varrho)]}_{\text{nonanalytic}}$$

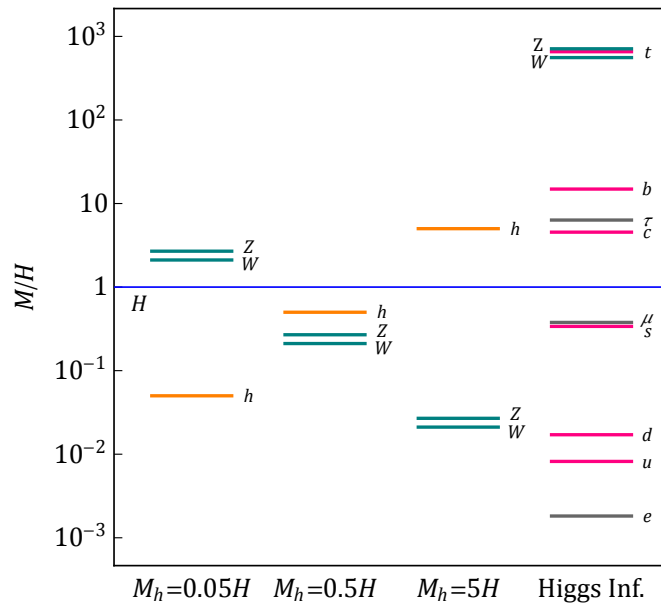
		B	L	α
$s = 0, m > \frac{3}{2}, \mu = 0$	tree	$\sim e^{-\pi m}$	$\frac{1}{2}$	$\sqrt{m^2 - \frac{9}{4}}$
$s = 0, 0 < m < \frac{3}{2}, \mu = 0$	tree	-	$\frac{1}{2} - \sqrt{\frac{9}{4} - m^2}$	0
$s > 0, m > s - \frac{1}{2}, \mu = 0$	tree	$\sim e^{-\pi m}$	$\frac{1}{2}$	$\sqrt{m^2 - (s - \frac{1}{2})^2}$
$s > 0, 0 < m < s - \frac{1}{2}, \mu = 0$	tree	-	$\frac{1}{2} - \sqrt{m^2 - (s - \frac{1}{2})^2}$	0
$s = 0, m > \frac{3}{2}, \mu = 0$	1-loop	$e^{-2\pi m}$	2	$2\sqrt{m^2 - \frac{9}{4}}$
Dirac fermion, $m > 0, \mu = 0$	1-loop	$e^{-2\pi m}$	3	$2m$
Dirac fermion, $m > 0, \mu \geq 0$	1-loop	$e^{2\pi\mu - 2\pi\sqrt{m^2 + \mu^2}}$	2	$2\sqrt{m^2 + \mu^2}$
$s = 1, m > \frac{1}{2}, \mu \geq 0$	1-loop	$e^{2\pi\mu - 2\pi m}$	2	$2\sqrt{m^2 - \frac{1}{4}}$

Phenomenology

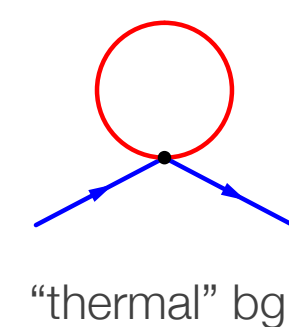
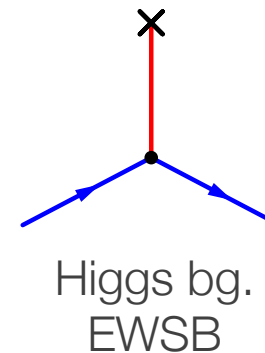
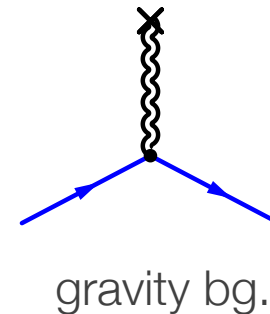
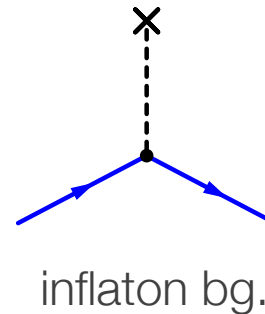
Quantum fields during inflation can receive various mass corrections

What you measured at the cosmological collider is the dressed mass

An example: The SM spectrum in inflation



Chen, Wang, ZZX, 1604.07841,
1610.06597, 1612.08122



Phenomenology

Signal size usually tiny in minimal models

Slow-roll inflaton; Scale invariance; O(1) coupling; No tuning

$$\frac{1}{\Lambda^2} (\partial_\mu \phi)^2 \sigma^2 \longrightarrow \frac{\dot{\phi}_0^2}{\Lambda^2} \sim H^2 \longrightarrow \Lambda \simeq 3600 H$$

No Boltzmann suppression

$\dot{\phi}_0 \simeq (60H)^2$

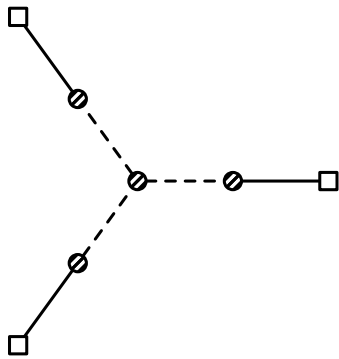
$$\mathcal{L} = \frac{1}{2} (\partial_\mu \sigma)^2 - \frac{1}{2} m^2 \sigma^2 - \lambda \sigma^4 + \frac{1}{\Lambda^2} (\partial_\mu \phi)^2 \sigma^2$$

$$f_{\text{NL}} \sim 3600 \cdot \left(\frac{\dot{\phi}_0}{\Lambda^2} \langle \sigma \rangle \right)^3 \cdot \lambda \langle \sigma \rangle \sim 10^{-7} \cdot \lambda \langle \sigma \rangle^4$$

QSFI: a very shallow potential is needed

$$\langle \sigma \rangle^2 \sim H^2 / \lambda \longrightarrow \lambda \langle \sigma \rangle^4 \sim 1 / \lambda \longrightarrow \lambda \lesssim 10^{-7}$$

$f_{\text{NL}} \gtrsim 1$



Phenomenology

$$\frac{1}{\Lambda}(\partial_\mu\phi)\mathcal{J}^\mu \longrightarrow \frac{1}{\Lambda}\dot{\phi}_0\mathcal{N}$$

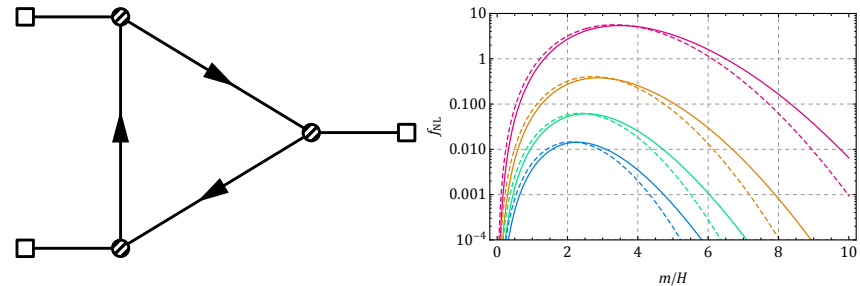
A new source of
particle production

Lian-Tao Wang, ZZX, 1910.12876

Fermion $(\partial_\mu\phi)\bar{\Psi}\gamma^\mu\gamma^5\Psi$

Probing heavy neutrinos

Chen, Wang, ZZX, 1805.02656

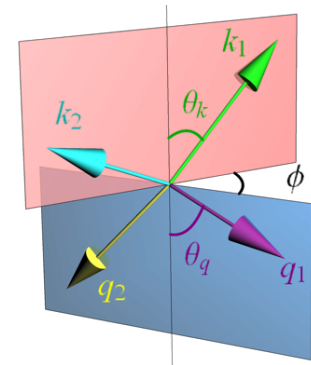
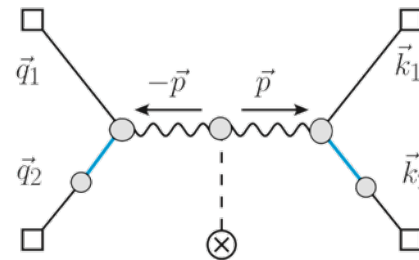


Gauge boson $\phi F\tilde{F}$

Lian-Tao Wang, ZZX, 2004.02887

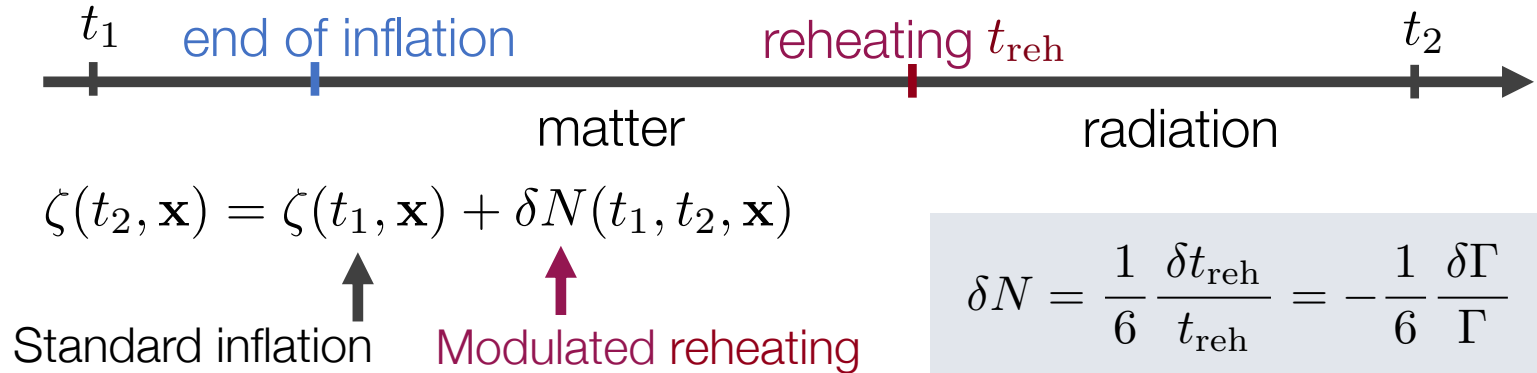
CP-breaking in trispectrum

Liu, Tong, Wang, ZZX, 1909.01819

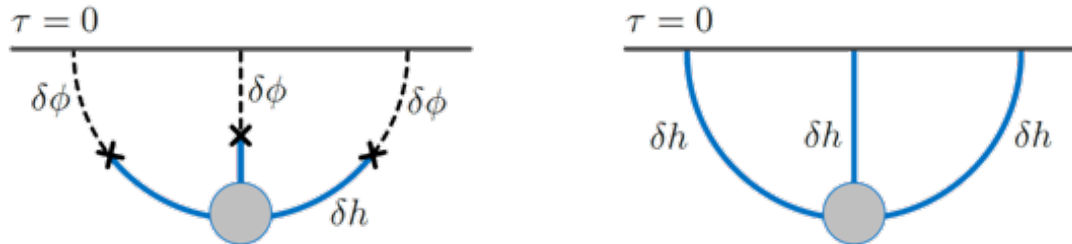


Phenomenology

Beyond slow-roll inflation



Modulated reheating: scalar perturbation generated not by the inflaton, but by a light field that modulates the inflaton decay



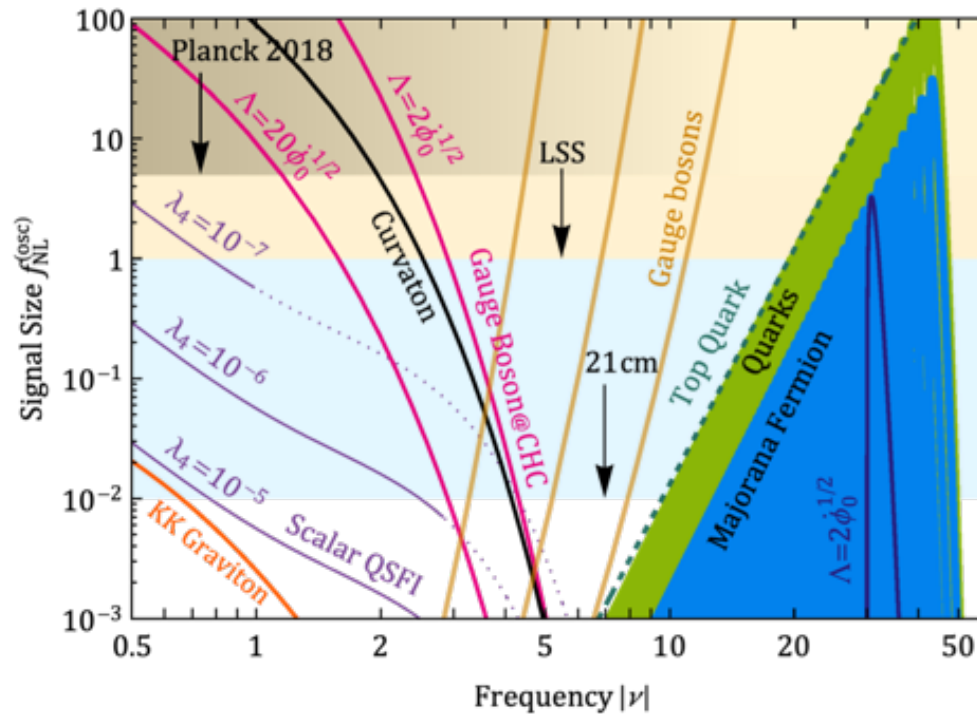
SM Higgs can do it $\rightarrow$ A cosmological Higgs collider

Large couplings / large signal / much less free parameters

Lu, Wang, ZZX, 1907.07390

Phenomenology

A status summary: known particle models producing observably large signals without tuning parameters



Lian-Tao Wang, ZZX, 1910.12876, 2004.02887

Final remarks

The non-G is from **interactions**
It must be there since fields must interact, **at least gravitationally**
A guaranteed signal at 0.01 level

The oscillatory non-G is largely a **kinematic** property
The signal is quite **inflation-model-independent**
but are sensitive to how particles couple to long-lived scalar mode

We are still at a very preliminary stage in understanding
& calculating cosmic correlators
A theory challenge

Observation data ahead!
But we are still very far from exhausting interesting
phenomenological possibilities @ cosmological collider
More efforts from particle physics are called for!